Assignment

Derivative at a Point

Basic Level

1. If $f(x) = |x|$, then $f'(0) =$ **[MNR 1982]**

(a) 0 (b) 1 (c) x (d) None of these

2. If $f(x) = \begin{cases} 1, & x < 0 \\ 1 + \sin x, & 0 \le x < \dfrac{\pi}{2} \end{cases}$ then $f'(0) =$ **[MP PET 1994]**

(a) 1 (b) 0 (c) ∞ (d) Does not exist

3. If $f(x) = \begin{cases} ax^2 + b; & x \le 0 \\ x^2; & x > 0 \end{cases}$ possesses derivative at $x = 0$, then

(a) $a = 0, b = 0$ (b) $a > 0, b = 0$ (c) $a \in R, b = 0$ (d) None of these

4. The derivative of $f(x) = 3|2 + x|$ at the point $x_0 = -3$ is **[Orissa JEE 2002]**

(a) 3 (b) – 3 (c) 0 (d) Does not exist

5. The derivative of $y = 1 - |x|$ at $x = 0$ is **[SCRA 1996]**

(a) 0 (b) 1 (c) – 1 (d) Does not exist

6. The derivative of $f(x) = |x^2 - x|$ at $x = 2$ is **[AMU 1999]**

(a) – 3 (b) 0 (c) 3 (d) Not defined

7. The value of $\dfrac{d}{dx}[|x-1| + |x-5|]$ at $x = 3$ is **[MP PET 2000]**

(a) – 2 (b) 0 (c) 2 (d) 4

8. If $f(x)$ has a derivative at $x = a$, then $\lim_{x \to a} \dfrac{xf(a) - af(x)}{x - a}$ is equal to

(a) $f(a) - af'(a)$ (b) $af(a) - f'(a)$ (c) $f(a) + f'(a)$ (d) $af(a) + f'(a)$

9. If $f(x) = x + 2$, then $f'(f(x))$ at $x = 4$ is **[DCE 2001]**

(a) 8 (b) 1 (c) 4 (d) 5

10. Let $3f(x) - 2f(1/x) = x$, then $f'(2)$ is equal to **[MP PET 2000]**

(a) 2/7 (b) 1/2 (c) 2 (d) 7/2

11. If $f(x)$ is a differentiable function, then $\lim_{x \to a} \dfrac{af(x) - xf(a)}{x - a}$ is

(a) $af'(a) - f(a)$ (b) $af(a) - f'(a)$ (c) $af'(a) + f(a)$ (d) $af(a) + f'(a)$

12. The differential coefficient of the function $|x - 1| + |x - 3|$ at the point $x = 2$ is **[Rajasthan PET 2002]**

(a) – 2 (b) 0 (c) 2 (d) Undefined

13. If $f(x) = |x - 3|$, then $f'(3) =$

(a) 0 (b) 1 (c) –1 (d) Does not exist

Advance Level

14. If $y = \cot^{-1}(\cos 2x)^{1/2}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ will be

(a) $\left(\frac{2}{3}\right)^{1/2}$ (b) $\left(\frac{1}{3}\right)^{1/2}$ (c) $(3)^{1/2}$ (d) $(6)^{1/2}$

15. The values of x, at which the first derivative of the function $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ *w.r.t.* x is $\frac{3}{4}$, are

(a) ± 2 (b) $\pm\frac{1}{2}$ (c) $\pm\frac{\sqrt{3}}{2}$ (d) $\pm\frac{2}{\sqrt{3}}$

16. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval (0, 2), is

[MNR 1995]

(a) 1 (b) 2 (c) 3 (d) 4

17. The set of all those points, where the function $f(x) = \frac{x}{1 + |x|}$ is differentiable, is

(a) $(-\infty, \infty)$ (b) $[0, \infty)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(0, \infty)$

18. Let $f(x+y) = f(x)f(y)$ and $f(x) = 1 + xg(x)G(x)$ where $\lim_{x\to 0} g(x) = a$ and $\lim_{x\to 0} G(x) = b$ then $f'(x)$ is equal to

(a) 1+ *ab* (b) *ab* (c) *a*/*b* (d) None of these

19. $f(x)$ is a function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$ and $h(x)$ is a function such that $h(x) = [f(x)]^2 + [g(x)]^2$ and $h(5) = 11$, then the value of $h(10)$ is

(a) 0 (b) 1 (c) 10 (d) None of these

20. Let $f(x+y) = f(x)f(y)$ for all x and y. Suppose that $f(3) = 3$ and $f'(0) = 11$, then $f'(3)$ is given by

(a) 22 (b) 33 (c) 28 (d) None of these

Some Standard Differentiation

Basic Level

21. If $y = \frac{(1-x)^2}{x^2}$, then $\frac{dy}{dx}$ is **[MP PET 1999]**

(a) $\frac{2}{x^2} + \frac{2}{x^3}$ (b) $-\frac{2}{x^2} + \frac{2}{x^3}$ (c) $-\frac{2}{x^2} - \frac{2}{x^3}$ (d) $-\frac{2}{x^3} + \frac{2}{x^2}$

22. If $2t = v^2$, then $\frac{dv}{dt}$ is equal to **[MP PET 1992]**

(a) 0 (b) 1/4 (c) 1/2 (d) 1/*v*

23. If $x = y\sqrt{1-y^2}$, then $\frac{dy}{dx} =$ **[MP PET 2001]**

(a) 0 (b) x (c) $\frac{\sqrt{1-y^2}}{1-2y^2}$ (d) $\frac{\sqrt{1-y^2}}{1+2y^2}$

24. If *pv* = 81, then $\frac{dp}{dv}$ is at *v* = 9 equal to **[MP PET 1999]**

(a) 1 (b) –1 (c) 2 (d) None of these

25. If $y=\sqrt{\frac{1+x}{1-x}}$, then $\frac{dy}{dx}=$ **[AISSE 1981; Rajasthan PET 1995]**

(a) $\frac{2}{(1+x)^{1/2}(1-x)^{3/2}}$ (b) $\frac{1}{(1+x)^{1/2}(1-x)^{3/2}}$ (c) $\frac{1}{2(1+x)^{1/2}(1-x)^{3/2}}$ (d) $\frac{2}{(1+x)^{3/2}(1-x)^{1/2}}$

26. The derivative of $f(x)=x|x|$ is

(a) $2x$ (b) $-2x$ (c) $2x^2$ (d) $2|x|$

27. The derivative of $F[f\{\phi(x)\}]$ is **[AMU 2001]**

(a) $F'[f\{\phi(x)\}]$ (b) $F'[f\{\phi(x)\}]f'\{\phi(x)\}$ (c) $F'[f\{\phi(x)\}]f'\{\phi(x)\}$ (d) $F'[f\{\phi(x)\}]f'\{\phi(x)\}\phi'(x)$

28. $\frac{d}{dx}(\sin 2x^2)$ equals **[Rajasthan PET 1996]**

(a) $4x\cos(2x^2)$ (b) $2\sin x^2.\cos x^2$ (c) $4x\sin(x^2)$ (d) $4x\sin(x^2).\cos(x^2)$

29. If $y=\sec x^0$, then $\frac{dy}{dx}=$ **[MP PET 1997]**

(a) $\sec x\tan x$ (b) $\sec x^0\tan x^0$ (c) $\frac{\pi}{180}\sec x^0\tan x^0$ (d) $\frac{180}{\pi}\sec x^0\tan x^0$

30. If $\sin y+e^{-x\cos y}=e$, then $\frac{dy}{dx}$ at $(1,\pi)$ is **[Kerala (Engg.) 2002]**

(a) $\sin y$ (b) $-x\cos y$ (c) e (d) $\sin y-x\cos y$

31. If $y=a\sin x+b\cos x$, then $y^2+\left(\frac{dy}{dx}\right)^2$ is a

(a) Function of x (b) Function of y (c) Function of x and y (d) Constant

32. $\frac{d}{dx}[\cos(1-x^2)^2]=$ **[AISSE 1981; AI CBSE 1979]**

(a) $-2x(1-x^2)\sin(1-x^2)^2$ (b) $-4x(1-x^2)\sin(1-x^2)^2$ (c) $4x(1-x^2)\sin(1-x^2)^2$ (d) $-2(1-x^2)\sin(1-x^2)^2$

33. If $y=\cos(\sin x^2)$, then at $x=\sqrt{\frac{\pi}{2}}, \frac{dy}{dx}=$

(a) -2 (b) 2 (c) $-2\sqrt{\frac{\pi}{2}}$ (d) 0

34. $\frac{d}{dx}[\sin^n x\cos nx]=$

(a) $n\sin^{n-1}x\cos(n+1)x$ (b) $n\sin^{n-1}x\cos nx$ (c) $n\sin^{n-1}x\cos(n-1)x$ (d) $n\sin^{n-1}x\sin(n+1)x$

35. $\frac{d}{dx}\cos(\sin x^2)=$ **[DSSE 1979]**

(a) $\sin(\sin x^2).\cos x^2.2x$ (b) $-\sin(\sin x^2).\cos x^2.2x$ (c) $-\sin(\sin x^2).\cos^2 x.2x$ (d) None of these

36. If $y=\sin(\sqrt{\sin x+\cos x})$, then $\frac{dy}{dx}=$ **[DSSE 1987]**

(a) $\frac{1}{2}\frac{\cos\sqrt{\sin x+\cos x}}{\sqrt{\sin x+\cos x}}$ (b) $\frac{\cos\sqrt{\sin x+\cos x}}{\sqrt{\sin x+\cos x}}$

(c) $\frac{1}{2}\frac{\cos\sqrt{\sin x+\cos x}}{\sqrt{\sin x+\cos x}}(\cos x-\sin x)$ (d) None of these

37. If $y=\sin\left(\frac{1+x^2}{1-x^2}\right)$, then $\frac{dy}{dx}=$ **[AISSE 1987]**

(a) $\frac{4x}{1-x^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$ (b) $\frac{x}{(1-x^2)^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$ (c) $\frac{x}{(1-x^2)}.\cos\left(\frac{1+x^2}{1-x^2}\right)$ (d) $\frac{4x}{(1-x^2)^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$

38. $\frac{d}{dx}(x^2+\cos x)^4=$ **[DSSE 1987]**

(a) $4(x^2+\cos x)(2x-\sin x)$ (b) $4(x^2-\cos x)(2x-\sin x)$ (c) $4(x^2+\cos x)^3(2x-\sin x)$ (d) $4(x^2+\cos x)^3(2x+\sin x)$

39. $\frac{d}{dx}\left(\frac{\cot^2 x-1}{\cot^2 x+1}\right)=$

(a) – sin $2x$ (b) 2 sin $2x$ (c) 2 cos $2x$ (d) – 2 sin $2x$

40. $\frac{d}{dx}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}=$ **[AISSE 1985; DSSE 1986]**

(a) $\sec^2 x$ (b) $-\sec^2\left(\frac{\pi}{4}-x\right)$ (c) $\sec^2\left(\frac{\pi}{4}+x\right)$ (d) $\sec^2\left(\frac{\pi}{4}-x\right)$

41. If $y=\frac{\tan x+\cot x}{\tan x-\cot x}$, then $\frac{dy}{dx}=$

(a) 2 tan $2x$ sec $2x$ (b) tan $2x$ sec $2x$ (c) – tan $2x$ sec $2x$ (d) – 2 tan $2x$ sec $2x$

42. $\frac{d}{dx}\sqrt{\sec^2 x+\cos ec^2 x}=$ **[DSSE 1981]**

(a) 4 cosec $2x$. cot $2x$ (b) – 4 cosec $2x$. cot $2x$ (c) – 4 cosec x . cot $2x$ (d) None of these

43. If $y=\frac{5x}{\sqrt[3]{(1-x)^2}}+\cos^2(2x+1)$, then $\frac{dy}{dx}=$ **[IIT 1980]**

(a) $\frac{5(3-x)}{3(1-x)^{5/3}}-2\sin(4x+2)$ (b) $\frac{5(3-x)}{3(1-x)^{2/3}}-2\sin(4x+4)$ (c) $\frac{5(3-x)}{3(1-x)^{2/3}}-2\sin(2x+1)$ (d) None of these

44. If $y=\sqrt{\sin x+y}$, then $\frac{dy}{dx}$ equals to **[Rajasthan PET 2001]**

(a) $\frac{\sin x}{2y-1}$ (b) $\frac{\cos x}{2y-1}$ (c) $\frac{\sin x}{2y+1}$ (d) $\frac{\cos x}{2y+1}$

45. $\frac{d}{dx}\log|x|=.....(x\neq 0)$

(a) $\frac{1}{x}$ (b) $-\frac{1}{x}$ (c) x (d) – x

46. $\frac{d}{dx}\log_{\sqrt{x}}(1/x)$ is equal to **[AMU 1999]**

(a) $-\frac{1}{2\sqrt{x}}$ (b) – 2 (c) $-\frac{1}{x^2\sqrt{x}}$ (d) 0

47. $\frac{d}{dx}\log(\log x)=$ **[IIT 1985]**

(a) $\frac{x}{\log x}$ (b) $\frac{\log x}{x}$ (c) $(x\log x)^{-1}$ (d) None of these

48. $\frac{d}{dx}(\log\tan x)=$ **[MNR 1986]**

(a) 2 sec $2x$ (b) 2 cosec $2x$ (c) sec $2x$ (d) cosec $2x$

49. If $y=\log x^x$, then $\frac{dy}{dx}=$ **[MNR 1978]**

(a) $x^x(1+\log x)$ (b) log (ex) (c) $\log\left(\frac{e}{x}\right)$ (d) None of these

50. Derivative of the function $f(x)=\log_5(\log_7 x), x>7$ is **[Orissa JEE 2002]**

(a) $\frac{1}{x(\ln 5)(\ln 7)(\log_7 x)}$ (b) $\frac{1}{x(\ln 5)(\ln 7)}$ (c) $\frac{1}{x(\ln x)}$ (d) None of these

51. The differential coefficient of $f[\log(x)]$ when $f(x)=\log x$ is **[Kurukshetra CEE 1998; DCE 2000]**

(a) $x\log x$ (b) $\frac{x}{\log x}$ (c) $\frac{1}{x\log x}$ (d) $\frac{\log x}{x}$

52. If $y=\log\frac{1+\sqrt{x}}{1-\sqrt{x}}$, then $\frac{dy}{dx}=$

(a) $\frac{\sqrt{x}}{1-x}$ (b) $\frac{1}{\sqrt{x}(1-x)}$ (c) $\frac{\sqrt{x}}{1+x}$ (d) $\frac{1}{\sqrt{x}(1+x)}$

53. If $y=x^2\log x+\frac{2}{\sqrt{x}}$, then $\frac{dy}{dx}=$

(a) $x+2x\log x-\frac{1}{\sqrt{x}}$ (b) $x+2x\log x-\frac{1}{x^{3/2}}$ (c) $x+2x\log x-\frac{2}{x^{3/2}}$ (d) None of these

54. $\frac{d}{dx}\left[\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right]=$ **[BIT Ranchi 1990]**

(a) sec x (b) cosec x (c) cosec $\frac{x}{2}$ (d) $\sec\frac{x}{2}$

55. $\frac{d}{dx}\left\{\log\left(\frac{e^x}{1+e^x}\right)\right\}=$

(a) $\frac{1}{1-e^x}$ (b) $-\frac{1}{1+e^x}$ (c) $-\frac{1}{1-e^x}$ (d) None of these

56. $\frac{d}{dx}\{\log(\sec x+\tan x)\}=$ **[AISSE 1982]**

(a) cos x (b) sec x (c) tan x (d) cot x

57. $\frac{d}{dx}\left[\log\left(x+\frac{1}{x}\right)\right]=$ **[MP PET 1995]**

(a) $\left(x+\frac{1}{x}\right)$ (b) $\frac{\left(1+\frac{1}{x^2}\right)}{\left(1+\frac{1}{x}\right)}$ (c) $\frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)}$ (d) $\left(1+\frac{1}{x}\right)$

58. $\frac{d}{dx}\log(x^{10})=$ **[Rajasthan PET 1992]**

(a) x^{-10} (b) $10x$ (c) $10/x$ (d) $10x^9$

59. If $y=\log\left\{\frac{x+\sqrt{(a^2+x^2)}}{a}\right\}$, then the value of $\frac{dy}{dx}$ is

(a) $\sqrt{a^2-x^2}$ (b) $a\sqrt{a^2+x^2}$ (c) $\frac{1}{\sqrt{a^2+x^2}}$ (d) $x\sqrt{a^2+x^2}$

60. If $y = \log(\sin^{-1} x)$, then $\frac{dy}{dx}$ equals

(a) $\frac{1}{\sin^{-1} x\sqrt{1-x^2}}$ (b) $-\frac{1}{\sin^{-1} x\sqrt{1-x^2}}$ (c) $\frac{-2x}{\sin^{-1} x\sqrt{1-x^2}}$ (d) None of these

61. If $y = e^{(1+\log_e x)}$, then the value of $\frac{dy}{dx} =$ **[MP PET 1996]**

(a) e (b) 1 (c) 0 (d) $\log_e xe^{\log_e ex}$

62. If $y = e^{\sqrt{x}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (b) $\frac{\sqrt{x}}{e^{\sqrt{x}}}$ (c) $\frac{x}{e^{\sqrt{x}}}$ (d) $\frac{2\sqrt{x}}{e^{\sqrt{x}}}$

63. The derivative of $y = x^{\ln x}$ is

(a) $x^{\ln x} \ln x$ (b) $x^{\ln x - 1} \ln x$ (c) $2x^{\ln x - 1} \ln x$ (d) $x^{\ln x - 2}$

64. Derivative of $x^6 + 6^x$ with respect to x is **[Kerala (Engg.) 2002]**

(a) $12x$ (b) $x + 4$ (c) $6x^5 + 6^x \log 6$ (d) $6x^5 + x6^{x-1}$

65. $\frac{d}{dx}(e^x \log \sin 2x) =$ **[AI CBSE 1985]**

(a) $e^x(\log \sin 2x + 2 \cot 2x)$ (b) $e^x(\log \cos 2x + 2 \cot 2x)$ (c) $e^x(\log \cos 2x + \cot 2x)$ (d) None of these

66. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\infty$, then $\frac{dy}{dx} =$ **[Karnataka CET 1999]**

(a) y (b) $y - 1$ (c) $y + 1$ (d) None of these

67. $\frac{d}{dx} e^{x \sin x} =$ **[DSSE 1979]**

(a) $e^{x \sin x}(x \cos x + \sin x)$ (b) $e^{x \sin x}(\cos x + x \sin x)$ (c) $e^{x \sin x}(\cos x + \sin x)$ (d) None of these

68. $\frac{d}{dx}(xe^{x^2}) =$ **[DSSE 1981]**

(a) $2x^2 e^{x^2} + e^{x^2}$ (b) $x^2 e^{x^2} + e^{x^2}$ (c) $e^x . 2x^2 + e^{x^2}$ (d) None of these

69. If $y = x^2 + x^{\log x}$, then $\frac{dy}{dx} =$

(a) $\frac{x^2 + \log x . x^{\log x}}{x}$ (b) $x^2 + \log x . x^{\log x}$ (c) $\frac{2(x^2 + \log x . x^{\log x})}{x}$ (d) None of these

70. $\frac{d}{dx}\{e^{-ax^2} \log(\sin x)\} =$ **[AI CBSE 1984]**

(a) $e^{-ax^2}(\cot x + 2ax \log \sin x)$ (b) $e^{-ax^2}(\cot x + ax \log \sin x)$

(c) $e^{-ax^2}(\cot x - 2ax \log \sin x)$ (d) None of these

71. If $y = \sqrt{\frac{1+e^x}{1-e^x}}$, then $\frac{dy}{dx} =$ **[AI CBSE 1987]**

(a) $\frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$ (b) $\frac{e^x}{(1-e^x)\sqrt{1-e^x}}$ (c) $\frac{e^x}{(1-e^x)\sqrt{1+e^{2x}}}$ (d) $\frac{e^x}{(1-e^x)\sqrt{1+e^x}}$

72. $\frac{d}{dx}\{e^x \log(1+x^2)\} =$ **[AI CBSE 1987]**

(a) $e^x\left[\log(1+x^2)+\frac{2x}{1+x^2}\right]$ (b) $e^x\left[\log(1+x^2)-\frac{2x}{1+x^2}\right]$ (c) $e^x\left[\log(1+x^2)+\frac{x}{1+x^2}\right]$ (d) $e^x\left[\log(1+x^2)-\frac{x}{1+x^2}\right]$

73. If $y=\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$, then $\frac{dy}{dx}=$

(a) $\frac{-8}{(e^{2x}-e^{-2x})^2}$ (b) $\frac{8}{(e^{2x}-e^{-2x})^2}$ (c) $\frac{-4}{(e^{2x}-e^{-2x})^2}$ (d) $\frac{4}{(e^{2x}-e^{-2x})^2}$

74. $\frac{d}{dx}(e^{x^3})$ is equal to **[Rajasthan PET 1995]**

(a) $3xe^{x^3}$ (b) $3x^2e^{x^3}$ (c) $3x(e^{x^3})^2$ (d) $2x^3e^{x^3}$

75. $\frac{d}{dx}[e^{ax}\cos(bx+c)]=$ **[AISSE 1989]**

(a) $e^{ax}[a\cos(bx+c)-b\sin(bx+c)]$ (b) $e^{ax}[a\sin(bx+c)-b\cos(bx+c)]$

(c) $e^{ax}[\cos(bx+c)-\sin(bx+c)]$ (d) None of these

76. If $y=e^x\log x$, then $\frac{dy}{dx}$ is **[SCRA 1996]**

(a) $\frac{e^x}{x}$ (b) $e^x\left(\frac{1}{x}+x\log x\right)$ (c) $e^x\left(\frac{1}{x}+\log x\right)$ (d) $\frac{e^x}{\log x}$

77. If $f(1)=3$, $f'(1)=2$, then $\frac{d}{dx}\{\log f(e^x+2x)\}$ at x = 0 is **[AMU 1999]**

(a) 2/3 (b) 3/2 (c) 2 (d) 0

78. $\frac{d}{dx}(\sin^{-1}x)$ is equal to **[Rajasthan PET 1995]**

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $-\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{1}{\sqrt{x^2-1}}$

79. If $y=\sin^{-1}\sqrt{x}$, then $\frac{dy}{dx}=$ **[MP PET 1995]**

(a) $\frac{2}{\sqrt{x}\sqrt{1-x}}$ (b) $\frac{-2}{\sqrt{x}\sqrt{1-x}}$ (c) $\frac{1}{2\sqrt{x}\sqrt{1-x}}$ (d) $\frac{1}{\sqrt{1-x}}$

80. If $y=\sin^{-1}\sqrt{1-x^2}$, then $\frac{dy}{dx}=$ **[AISSE 1987]**

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1+x^2}}$ (c) $-\frac{1}{\sqrt{1-x^2}}$ (d) $-\frac{1}{\sqrt{x^2-1}}$

81. $\frac{d}{dx}\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}=$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) – 1

82. If $y=\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$, then $y'(1)$ is **[AMU 2000]**

(a) 0 (b) $\frac{1}{2}$ (c) – 1 (d) $-\frac{1}{4}$

83. Differential coefficient of $\sec^{-1} x$ is

(a) $\frac{1}{x\sqrt{1-x^2}}$ (b) $-\frac{1}{x\sqrt{1-x^2}}$ (c) $\frac{1}{x\sqrt{x^2-1}}$ (d) $-\frac{1}{x\sqrt{x^2-1}}$

84. If $y = \cot^{-1}\left(\frac{1+x}{1-x}\right)$, then $\frac{dy}{dx} =$ **[DSSE 1984]**

(a) $\frac{1}{1+x^2}$ (b) $-\frac{1}{1+x^2}$ (c) $\frac{2}{1+x^2}$ (d) $-\frac{2}{1+x^2}$

85. If $y = \tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$, then $\frac{dy}{dx}$ is equal to **[Roorkee 1995]**

(a) 0 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 1

86. If $f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$, then $f'\left(\frac{\pi}{3}\right) =$ **[BIT Ranchi 1988]**

(a) $\frac{1}{2(1+\cos x)}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) None of these

87. If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx} =$ **[MNR 1984]**

(a) 0 (b) 1 (c) 2 (d) 3

88. If $y = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$, then $\frac{dy}{dx} =$

(a) $-\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{x}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\frac{\sqrt{1-x^2}}{x}$

89. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1996; EAMCET 1991]**

(a) $\frac{2}{1-x^2}$ (b) $\frac{1}{1+x^2}$ (c) $\pm\frac{2}{1+x^2}$ (d) $-\frac{2}{1+x^2}$

90. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right] =$ **[Karnataka CET 2001]**

(a) $-\frac{1}{1+x^2}$ (b) $\frac{1}{1+a^2} - \frac{1}{1+x^2}$ (c) $\frac{1}{1+\left(\frac{a-x}{1+ax}\right)^2}$ (d) $\frac{-1}{\sqrt{1-\left(\frac{a-x}{1+ax}\right)^2}}$

91. If $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right]$, then $\frac{dy}{dx}$ is **[UPSEAT 2001]**

(a) 1/2 (b) $\pi/4$ (c) 0 (d) 1

92. $\frac{d}{dx}\left(\tan^{-1}\frac{\cos x}{1+\sin x}\right) =$ **[AISSE 1984, 85; MNR 1983; Rajasthan PET 1994, 96]**

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) – 1 (d) 1

93. If $y = \sin^{-1}\frac{2x}{1+x^2} + \sec^{-1}\frac{1+x^2}{1-x^2}$, then $\frac{dy}{dx} =$ **[Rajasthan PET 1996]**

(a) $\frac{4}{1-x^2}$ (b) $\frac{1}{1+x^2}$ (c) $\frac{4}{1+x^2}$ (d) $\frac{-4}{1+x^2}$

94. If $y=\tan^{-1}\frac{4x}{1+5x^2}+\tan^{-1}\frac{2+3x}{3-2x}$, then $\frac{dy}{dx}=$

(a) $\frac{1}{1+25x^2}+\frac{2}{1+x^2}$ (b) $\frac{5}{1+25x^2}+\frac{2}{1+x^2}$ (c) $\frac{5}{1+25x^2}$ (d) $\frac{1}{1+25x^2}$

95. $\frac{d}{dx}\sin^{-1}\left(\frac{x^2}{\sqrt{x^4+a^4}}\right)=$

(a) $\frac{2a^4x}{a^4+x^4}$ (b) $\frac{2a^2x}{a^4+x^4}$ (c) $\frac{2a^3x}{a^4+x^4}$ (d) $\frac{-2a^2x}{a^4+x^4}$

96. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then

(a) $(1-x^2)\frac{dy}{dx}-xy-1=0$ (b) $(1-x^2)\frac{dy}{dx}+xy-1=0$ (c) $(1-x^2)\frac{dy}{dx}+\frac{1}{2}xy-1=0$ (d) None of these

97. $\frac{d}{dx}\sin^{-1}(3x-4x^3)=$ **[Rajasthan PET 2003]**

(a) $\frac{3}{\sqrt{1-x^2}}$ (b) $\frac{-3}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\frac{-1}{\sqrt{1-x^2}}$

98. $\frac{d}{dx}\sin^{-1}(2ax\sqrt{1-a^2x^2})=$

(a) $\frac{2a}{\sqrt{a^2-x^2}}$ (b) $\frac{a}{\sqrt{a^2-x^2}}$ (c) $\frac{2a}{\sqrt{1-a^2x^2}}$ (d) $\frac{a}{\sqrt{1-a^2x^2}}$

99. $\frac{d}{dx}\left[\sin^{-1}\left(\frac{3x}{2}-\frac{x^3}{2}\right)\right]$ equals

(a) $\frac{3}{\sqrt{4-x^2}}$ (b) $\frac{-3}{\sqrt{4-x^2}}$ (c) $\frac{1}{\sqrt{4-x^2}}$ (d) $\frac{-1}{\sqrt{4-x^2}}$

100. If $y=\sin^{-1}\left\{\frac{\sin x+\cos x}{\sqrt{2}}\right\}$, then $\frac{dy}{dx}=$

(a) 1 (b) – 1 (c) 0 (d) None of these

101. $\frac{d}{dx}\cos^{-1}\frac{x-x^{-1}}{x+x^{-1}}=$ **[DSSE 1985; Roorkee 1963]**

(a) $\frac{1}{1+x^2}$ (b) $-\frac{1}{1+x^2}$ (c) $\frac{2}{1+x^2}$ (d) $\frac{-2}{1+x^2}$

102. $\frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}=$ **[AISSE 1984]**

(a) $\frac{1}{1+x^2}$ (b) $-\frac{1}{1+x^2}$ (c) $-\frac{2}{1+x^2}$ (d) $\frac{2}{1+x^2}$

103. $\frac{d}{dx}\cos^{-1}\sqrt{\frac{1+x^2}{2}}=$ **[AI CBSE 1988]**

(a) $\frac{-1}{2\sqrt{1-x^4}}$ (b) $\frac{1}{2\sqrt{1-x^4}}$ (c) $\frac{-x}{\sqrt{1-x^4}}$ (d) $\frac{x}{\sqrt{1-x^4}}$

104. If $y=\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{x}}{1+\sqrt{ax}}\right)$, then $\frac{dy}{dx}=$ **[AI CBSE 1988]**

(a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{1}{(1+x)\sqrt{x}}$ (c) $-\frac{1}{2(1+x)\sqrt{x}}$ (d) None of these

105. $\frac{d}{dx}\left[\tan^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=$ **[Roorkee 1980]**

(a) $\frac{-x}{\sqrt{1-x^4}}$ (b) $\frac{x}{\sqrt{1-x^4}}$ (c) $\frac{-1}{2\sqrt{1-x^4}}$ (d) $\frac{1}{2\sqrt{1-x^4}}$

106. If $y=(1+x^2)\tan^{-1}x-x$, then $\frac{dy}{dx}=$ **[Karnataka CET 2001]**

(a) $\tan^{-1}x$ (b) $2x\tan^{-1}x$ (c) $2x\tan^{-1}x-1$ (d) $\frac{2x}{\tan^{-1}x}$

107. If $f(x)=(x+1)\tan^{-1}(e^{-2x})$, then $f'(0)$ equals

(a) $\frac{\pi}{6}+5$ (b) $\frac{\pi}{2}+1$ (c) $\frac{\pi}{4}-1$ (d) None of these

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108. If $y=\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ is **[DCE 2002; Haryana CEE 2001]**

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) $\frac{-x}{\sqrt{1+x^2}}$ (c) $\frac{x}{\sqrt{1-x^2}}$ (d) None of these

109. If $y=(1+x^{1/4})(1+x^{1/2})(1-x^{1/4})$, then $\frac{dy}{dx}=$ **[MP PET 1994]**

(a) 1 (b) – 1 (c) x (d) $\sqrt{x}$

110. If $f(x)=\sqrt{ax}+\frac{a^2}{\sqrt{ax}}$, then $f'(a)=$

(a) – 1 (b) 1 (c) 0 (d) a

111. $x\sqrt{1+y}+y\sqrt{1+x}=0$, then $\frac{dy}{dx}=$

(a) $1+x$ (b) $(1+x)^{-2}$ (c) $-(1+x)^{-1}$ (d) $-(1+x)^{-2}$

112. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then $\frac{dy}{dx}=$ **[MNR 1983; ISM Dhanbad 1987; Rajasthan PET 1991]**

(a) $\sqrt{\frac{1-x^2}{1-y^2}}$ (b) $\sqrt{\frac{1-y^2}{1-x^2}}$ (c) $\sqrt{\frac{x^2-1}{1-y^2}}$ (d) $\sqrt{\frac{y^2-1}{1-x^2}}$

113. Function $y=(x+\sqrt{x^2+1})^k$ satisfies **[IIT Screening]**

(a) $(x^2+1)y'=k^2y$ (b) $\sqrt{(x^2+1)}y'=ky$ (c) $(1+x^2)y''+ky'-xy=0$ (d) $(1+x^2)y''+k^2+xy'=0$

114. The derivative of $\sqrt{\sqrt{x}+1}$ is

(a) $\frac{1}{\sqrt{x}(\sqrt{x}+1)}$ (b) $\frac{-1}{\sqrt{x}\sqrt{x+1}}$ (c) $\frac{4}{\sqrt{x(\sqrt{x}+1)}}$ (d) $\frac{1}{4\sqrt{x(\sqrt{x}+1)}}$

115. If $f(x)=\frac{1}{\sqrt{x^2+a^2}+\sqrt{x^2+b^2}}$, then $f'(x)$ is equal to **[Kurukshetra CEE 1998]**

(a) $\frac{x}{(a^2-b^2)}\left[\frac{1}{\sqrt{x^2+a^2}}-\frac{1}{\sqrt{x^2+b^2}}\right]$

(b) $\frac{x}{(a^2+b^2)}\left[\frac{1}{\sqrt{x^2+a^2}}-\frac{1}{\sqrt{x^2+b^2}}\right]$

(c) $\frac{x}{(a^2-b^2)}\left[\frac{1}{\sqrt{x^2+a^2}}+\frac{1}{\sqrt{x^2+b^2}}\right]$

(d) $(a^2-b^2)\left[\frac{1}{\sqrt{x^2+a^2}}-\frac{2}{\sqrt{x^2+b^2}}\right]$

116. If $\sqrt{(1-x^6)}+\sqrt{(1-y^6)}=a^3(x^3-y^3)$, then $\frac{dy}{dx}=$ **[Roorkee 1994]**

(a) $\frac{x^2}{y^2}\sqrt{\frac{1-x^6}{1-y^6}}$ (b) $\frac{y^2}{x^2}\sqrt{\frac{1-y^6}{1-x^6}}$ (c) $\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$ (d) None of these

117. If $y=\sqrt{x+\sqrt{x}}$, then $y\frac{dy}{dx}$ equals

(a) $\frac{2\sqrt{x}+1}{4\sqrt{x}}$ (b) $\frac{\sqrt{x}+1}{2\sqrt{x}}$ (c) $\frac{\sqrt{x}+1}{4x}$ (d) $\frac{x+1}{2\sqrt{x}}$

118. If $y=\sqrt{\sin\sqrt{x}}$, then $\frac{dy}{dx}=$ **[MP PET 1997]**

(a) $\frac{1}{2\sqrt{\cos\sqrt{x}}}$ (b) $\frac{\sqrt{\cos\sqrt{x}}}{2x}$ (c) $\frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{\sin\sqrt{x}}}$ (d) $\frac{1}{2\sqrt{\sin x}}$

119. $\frac{d}{dx}\sqrt{x\sin x}=$ **[AISSE 1985]**

(a) $\frac{\sin x+x\cos x}{2\sqrt{x\sin x}}$ (b) $\frac{\sin x+x\cos x}{\sqrt{x\sin x}}$ (c) $\frac{x\sin x+\cos x}{\sqrt{2\sin x}}$ (d) $\frac{x\sin x+\cos x}{\sqrt{2x\sin x}}$

120. If $y=f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x)=\sin x^2$, then $\frac{dy}{dx}=$ **[IIT 1982]**

(a) $\frac{6x^2-2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

(b) $\frac{6x^2-2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$

(c) $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$

(d) $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

121. $\frac{d}{dx}\left[\frac{e^{ax}}{\sin(bx+c)}\right]=$ **[AI CBSE 1983]**

(a) $\frac{e^{ax}[a\sin(bx+c)+b\cos(bx+c)]}{\sin^2(bx+c)}$

(b) $\frac{e^{ax}[a\sin(bx+c)-b\cos(bx+c)]}{\sin^2(bx+c)}$

(c) $\frac{e^{ax}[a\sin(bx+c)-b\cos(bx+c)]}{\sin^2(bx+c)}$

(d) None of these

122. If $y=b\cos\log\left(\frac{x}{n}\right)^n$, then $\frac{dy}{dx}=$

(a) $-n\ b\sin\log\left(\frac{x}{n}\right)^b$ (b) $n\ b\sin\log\left(\frac{x}{n}\right)^n$ (c) $\frac{-nb}{x}\sin\log\left(\frac{x}{n}\right)^n$ (d) None of these

123. If $y=f\left(\frac{5x+1}{10x^2-3}\right)$ and $f'(x)=\cos x$, then $\frac{dy}{dx}=$ **[MP PET 1987]**

(a) $\cos\left(\frac{5x+1}{10x^2-3}\right)\frac{d}{dx}\left(\frac{5x+1}{10x^2-3}\right)$ (b) $\frac{5x+1}{10x^2-3}\cos\left(\frac{5x+1}{10x^2-3}\right)$

(c) $\cos\left(\frac{5x+1}{10x^2-3}\right)$ (d) None of these

124. $\frac{d}{dx}\left(x^3 \tan^2 \frac{x}{2}\right) =$ **[AISSE 1979]**

(a) $x^3 \tan \frac{x}{2} . \sec^2 \frac{x}{2} + 3x \tan^2 \frac{x}{2}$ (b) $x^3 \tan \frac{x}{2} . \sec^2 \frac{x}{2} + 3x^2 \tan^2 \frac{x}{2}$

(c) $x^2 \tan^2 \frac{x}{2} . \sec^2 \frac{x}{2} + 3x^2 \tan^2 \frac{x}{2}$ (d) None of these

125. $\frac{d}{dx}(\tan a^{1/x}) =$

(a) $\sec^2(a^{1/x}) \cdot \frac{(a^{1/x} . \log a)}{x^2}$ (b) $\sec^2(a^{1/x}).(a^{1/x}.\log a)$ (c) $\frac{\sec x . \log a}{x^2}$ (d) $-\frac{\sec^2(a^{1/x}).(a^{1/x}.\log_e a)}{x^2}$

126. If $y = \sqrt{\frac{1+\tan x}{1-\tan x}}$, then $\frac{dy}{dx} =$

(a) $\frac{1}{2}\sqrt{\frac{1-\tan x}{1+\tan x}} \cdot \sec^2\left(\frac{\pi}{4}+x\right)$ (b) $\sqrt{\frac{1-\tan x}{1+\tan x}} \cdot \sec^2\left(\frac{\pi}{4}+x\right)$ (c) $\frac{1}{2}\sqrt{\frac{1-\tan x}{1+\tan x}} \cdot \sec\left(\frac{\pi}{4}+x\right)$ (d)

127. If $A = \frac{2^x \cot x}{\sqrt{x}}$, then $\frac{dA}{dx} =$

(a) $\frac{2^{x-1}\left\{-2x \operatorname{cosec}^2 x + \cot x . \log\left(\frac{4^x}{e}\right)\right\}}{x^{3/2}}$ (b) $\frac{2^{x-1}\left\{-2x \operatorname{cosec}^2 x + \cot x . \log\left(\frac{4^x}{e}\right)\right\}}{x}$

(c) $\frac{2^{x}\left\{-2x \operatorname{cosec}^2 x + \cot x . \log\left(\frac{4^x}{e}\right)\right\}}{x^{3/2}}$ (d) None of these

128. Differential coefficient of $\sqrt{\sec \sqrt{x}}$ is **[MP PET 1996]**

(a) $\frac{1}{4\sqrt{x}}(\sec \sqrt{x})^{3/2} \sin \sqrt{x}$ (b) $\frac{1}{4\sqrt{x}} \sec \sqrt{x} \sin \sqrt{x}$ (c) $\frac{1}{2}\sqrt{x}(\sec \sqrt{x})^{3/2} \sin \sqrt{x}$ (d) $\frac{1}{2}\sqrt{x} \sec \sqrt{x} \sin \sqrt{x}$

129. $\frac{d}{dx}\left(\frac{\sec x + \tan x}{\sec x - \tan x}\right) =$ **[DSSE 1979, 81; CBSE 1981]**

(a) $\frac{2\cos x}{(1-\sin x)^2}$ (b) $\frac{\cos x}{(1-\sin x)^2}$ (c) $\frac{2\cos x}{1-\sin x}$ (d) None of these

130. If $x = f(m)\cos m - f'(m)\sin m$ and $y = f(m)\sin m + f'(m)\cos m$, then $\left(\frac{dy}{dm}\right)^2 + \left(\frac{dx}{dm}\right)^2$ equals **[AMU 1997]**

(a) $[f(m)+f''(m)]^2$ (b) $[f(m)-f''(m)]^2$ (c) $\{f(m)\}^2 + \{f'(m)\}^2$ (d) $\frac{\{f(m)\}^2}{\{f'(m)\}^2}$

131. If $x = \sec\theta - \cos\theta$ and $y = \sec^n \theta - \cos^n \theta$, then **[IIT 1989]**

(a) $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$ (b) $(x^2+4)\left(\frac{dy}{dx}\right)^2 = x^2(y^2+4)$ (c) $(x^2+4)\left(\frac{dy}{dx}\right)^2 = (y^2+4)$ (d)

132. If $y = \log_{\sin x}(\tan x)$, then $\left(\frac{dy}{dx}\right)_{\pi/4} =$

(a) $\frac{4}{\log 2}$ (b) – 4 log 2 (c) $\frac{-4}{\log 2}$ (d) None of these

133. If $u(x, y) = y \log x + x \log y$, then $u_x u_y - u_x \log x - u_y \log y + \log x \log y =$ **[EAMCET 2003]**

(a) 0 (b) – 1 (c) 1 (d) 2

134. If $y = \log x . e^{(\tan x + x^2)}$, then $\frac{dy}{dx} =$

(a) $e^{(\tan x+x^2)}\left[\frac{1}{x}+(\sec^2 x + x)\log x\right]$ (b) $e^{(\tan x+x^2)}\left[\frac{1}{x}+(\sec^2 x - x)\log x\right]$

(c) $e^{(\tan x+x^2)}\left[\frac{1}{x}+(\sec^2 x + 2x)\log x\right]$ (d) $e^{(\tan x+x^2)}\left[\frac{1}{x}+(\sec^2 x - 2x)\log x\right]$

135. $\frac{d}{dx}\left[\log \sqrt{\sin \sqrt{e^x}}\right] =$

(a) $\frac{1}{4}e^{x/2}\cot(e^{x/2})$ (b) $e^{x/2}\cot(e^{x/2})$ (c) $\frac{1}{4}e^{x}\cot(e^{x})$ (d) $\frac{1}{2}e^{x/2}\cot(e^{x/2})$

136. If $y\sqrt{x^2+1} = \log\{\sqrt{x^2+1} - x\}$, then $(x^2+1)\frac{dy}{dx} + xy + 1 =$ **[Roorkee 1978; Kurukshetra CEE 1998]**

(a) 0 (b) 1 (c) 2 (d) None of these

137. If $y = \log_{\cos x} \sin x$, then $\frac{dy}{dx}$ is equal to

(a) $(\cot x \log \cos x + \tan x \log \sin x)/(\log \cos x)^2$ (b) $(\tan x \log \cos x + \cot x \log \sin x)/(\log \cos x)^2$

(c) $(\cot x \log \cos x + \tan x \log \sin x)/(\log \sin x)^2$ (d) None of these

138. If $y = \log(x + e^{\sqrt{x}})$ then $\frac{dy}{dx} =$

(a) $\frac{2\sqrt{x}+e^{\sqrt{x}}}{2\sqrt{x}(x+e^{\sqrt{x}})}$ (b) $\frac{2\sqrt{x}+e^{\sqrt{x}}}{2\sqrt{x}(x-e^{\sqrt{x}})}$ (c) $\frac{2\sqrt{x}-e^{\sqrt{x}}}{2\sqrt{x}(x+e^{\sqrt{x}})}$ (d) $\frac{2\sqrt{x}-e^{\sqrt{x}}}{2\sqrt{x}(x-e^{\sqrt{x}})}$

139. $\frac{d}{dx}(a^{\log_{10} \operatorname{cosec}^{-1} x}) =$ **[Roorkee 1990]**

(a) $a^{\log_{10}\operatorname{cosec}^{-1}x} \cdot \frac{1}{\operatorname{cosec}^{-1}x} \cdot \frac{1}{x\sqrt{x^2-1}} \cdot \log_{10} a$ (b) $-a^{\log_{10}\operatorname{cosec}^{-1}x} \cdot \frac{1}{\operatorname{cosec}^{-1}x} \cdot \frac{1}{|x|\sqrt{x^2-1}} \cdot \log_{10} a$

(c) $a^{\log_{10}\operatorname{cosec}^{-1}x} \cdot \frac{1}{\operatorname{cosec}^{-1}x} \cdot \frac{1}{|x|\sqrt{x^2-1}} \cdot \log_{10} a$ (d) $-a^{\log_{10}\operatorname{cosec}^{-1}x} \cdot \frac{1}{\operatorname{cosec}^{-1}x} \cdot \frac{1}{x\sqrt{x^2-1}} \cdot \log_{10} a$

140. $\frac{d}{dx}(e^{\sqrt{1-x^2}} . \tan x) =$ **[AI CBSE 1979]**

(a) $e^{\sqrt{1-x^2}}\left[\sec^2 x + \frac{x \tan x}{\sqrt{1-x^2}}\right]$ (b) $e^{\sqrt{1-x^2}}\left[\sec^2 x - \frac{x \tan x}{\sqrt{1-x^2}}\right]$ (c) $e^{\sqrt{1-x^2}}\left[\sec^2 x + \frac{\tan x}{\sqrt{1-x^2}}\right]$ (d)

141. $10^{-x \tan x}\left[\frac{d}{dx}(10^{x \tan x})\right]$ is equal to

(a) $\tan x + x \sec^2 x$ (b) $\ln 10(\tan x + x \sec^2 x)$

(c) $\ln 10\left(\tan x + \frac{x}{\cos^2 x} + \tan x \sec x\right)$ (d) $x \tan x \ln 10$

142. If $y = \sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$, then $\frac{dy}{dx} =$

(a) $\frac{1}{2\sqrt{1-x^2}}$ (b) $\frac{-2x}{\sqrt{1-x^2}}$ (c) $\frac{-x}{\sqrt{1-x^2}}$ (d) $\frac{x}{\sqrt{1-x^2}}$

143. $\frac{d}{dx}\left(\cos^{-1}\sqrt{\frac{1+\cos x}{2}}\right) =$ **[AI CBSE 1982]**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) None of these

144. If $f(x) = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$, then the value of $f'(e) =$ **[Karnataka CET 1999]**

(a) 1 (b) $\frac{1}{e}$ (c) $\frac{2}{e}$ (d) $\frac{2}{e^2}$

145. If $y = \frac{1}{\sqrt{a^2-b^2}}\cos^{-1}\left[\frac{a\cos(x-\alpha)+b}{\theta}\right]$ where $\theta = a + b\cos(x-\alpha)$, then $\frac{dy}{dx} =$ **[Orissa JEE 2003]**

(a) $\frac{1}{\theta}$ (b) $\frac{2}{\theta}$ (c) $\frac{1}{\theta^2}$ (d) $\frac{2}{\theta^2}$

146. $\frac{d}{dx}\tan^{-1}(\sec x + \tan x) =$ **[AISSE 1985, 87; DSSE 1982, 84]**

(a) 1 (b) 1/2 (c) $\cos x$ (d) $\sec x$

147. $\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right] =$ **[Ranchi BIT 1989; Roorkee 1989; Rajasthan PET 1996]**

(a) $-\frac{1}{2}$ (b) 0 (c) $\frac{1}{2}$ (d) 1

148. If $y = \tan^{-1}\left(\frac{x^{1/3}+a^{1/3}}{1-x^{1/3}a^{1/3}}\right)$, then $\frac{dy}{dx} =$ **[DSSE 1986]**

(a) $\frac{1}{3x^{2/3}(1+x^{2/3})}$ (b) $\frac{a}{3x^{2/3}(1+x^{2/3})}$ (c) $-\frac{1}{3x^{2/3}(1+x^{2/3})}$ (d) $-\frac{a}{3x^{2/3}(1+x^{2/3})}$

149. The differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ is **[MP PET 2003]**

(a) $\sqrt{1-x^2}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{2\sqrt{1-x^2}}$ (d) x

150. $\frac{d}{dx}\left[\sin^2\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}\right]$ equals **[EAMCET 1996]**

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

151. If $f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$ then $\frac{dy}{dx}$ equals

(a) $\sin(\log x).\frac{1}{x \log x}$ (b) $\frac{12}{(3-2x)^2}\sin\left(\log\left(\frac{2x+3}{3-2x}\right)\right)$ (c) $\sin\left(\log\left(\frac{2x+3}{3-2x}\right)\right)$ (d) None of these

152. If $y=\tan^{-1}\left\{\frac{3a^2x-x^3}{a(a^2-3x^2)}\right\}$ then $\frac{dy}{dx}$ equals **[IIT 1969; Rajasthan PET 1998]**

(a) $\frac{3}{a^2+x^2}$ (b) $\frac{a}{a^2+x^2}$ (c) $\frac{3a}{a^2+x^2}$ (d) $\frac{3x}{a^2+x^2}$

153. If $y=\sinh^{-1}(\tan x)$, then the value of $\frac{dy}{dx}$ is

(a) sin x (b) cos x (c) sec x (d) cosec x

154. $\frac{d}{dx}[\sinh^{-1}x]^x$ equals

(a) $\frac{x}{\sqrt{1+x^2}.\sinh^{-1}x}+\log(\sinh^{-1}x)$ (b) $(\sinh^{-1}x)^{x-1}\frac{1}{\sqrt{1+x^2}}$

(c) $(\sinh^{-1}x)^x\left[\frac{x}{\sqrt{1+x^2}.\sinh^{-1}x}+\log(\sinh^{-1}x)\right]$ (d) None of these

155. If $y=\sin^{-1}(x\sqrt{1-x}+\sqrt{x}\sqrt{1-x^2})$, then $\frac{dy}{dx}=$ **[Roorkee 1981]**

(a) $\frac{-2x}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$ (b) $\frac{-1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$ (c) $\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$ (d) None of these

156. If $y=\tan^{-1}\frac{x}{1+\sqrt{1-x^2}}+\sin\left\{2\tan^{-1}\sqrt{\left(\frac{1-x}{1+x}\right)}\right\}$, then $\frac{dy}{dx}=$

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1-2x}{\sqrt{1-x^2}}$ (c) $\frac{1-2x}{2\sqrt{1-x^2}}$ (d) $\frac{1}{1+x^2}$

157. If $y=\cot^{-1}\left[\frac{\sqrt{1+x^2}+1}{x}\right]$ then $\frac{dy}{dx}=$

(a) $\frac{1}{2}.\frac{1}{1+x^2}$ (b) $\frac{1}{2}.\frac{1}{1-x^2}$ (c) $\frac{2}{1+x^2}$ (d) $\frac{2}{1-x^2}$

158. If $y=\tan^{-1}\left(\frac{2^{x+1}}{1-4^x}\right)$, then $\frac{dy}{dx}=$

(a) $\frac{2^{x+1}\log_e 2}{4^x}$ (b) $\frac{2^{x+1}\log_e 2}{1+4^x}$ (c) $\frac{2^{x+1}\log_e 2}{1-4^x}$ (d) $\frac{2^{x+1}\log_2 e}{1-4^x}$

159. If $y=\tan^{-1}\left(\frac{2.a^x}{1-a^{2x}}\right)$, then $\frac{dy}{dx}=$

(a) $\frac{2.a^x\log a}{1-a^{2x}}$ (b) $\frac{2.a^x\log a}{1+a^{2x}}$ (c) $2.a^x\log a$ (d) $\frac{2.a^x\log a}{a^{2x}-1}$

160. If $y=x.e^{\cos^{-1}}x+\sec(2x-1)$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1986]**

(a) $e^{\cos^{-1}x}\left(1-\frac{x}{\sqrt{1-x^2}}\right)+\sec(2x-1).\tan(2x-1)$

(b) $e^{\cos^{-1}x}\left(1-\frac{x}{\sqrt{1-x^2}}\right)-\sec(2x-1).\tan(2x-1)$

(c) $e^{\cos^{-1}x}\left(1-\frac{x}{\sqrt{1-x^2}}\right)+2\sec(2x-1).\tan(2x-1)$

(d) None of these

161. If $y=\tan^{-1}\left(\frac{a+b\tan x}{b-a\tan x}\right)$, then $\frac{dy}{dx}=$

(a) 1 (b) – 1 (c) x (d) $\frac{1}{1+x^2}$

Methods of differentiation

Basic Level

162. If $x^3+8xy+y^3=64$, then $\frac{dy}{dx}=$

(a) $-\frac{3x^2+8y}{8x+3y^2}$ (b) $\frac{3x^2+8y}{8x+3y^2}$ (c) $\frac{3x+8y^2}{8x^2+3y}$ (d) None of these

163. If $\sin^2 x+2\cos y+xy=0$, then $\frac{dy}{dx}=$ **[AI CBSE 1980]**

(a) $\frac{y+2\sin x}{2\sin y+x}$ (b) $\frac{y+\sin 2x}{2\sin y-x}$ (c) $\frac{y+2\sin x}{\sin y+x}$ (d) None of these

164. If $y\sec x+\tan x+x^2y=0$, then $\frac{dy}{dx}=$ **[DSSE 1981; CBSE 1981]**

(a) $\frac{2xy+\sec^2 x+y\sec x\tan x}{x^2+\sec x}$ (b) $-\frac{2xy+\sec^2 x+\sec x\tan x}{x^2+\sec x}$

(c) $-\frac{2xy+\sec^2 x+y\sec x\tan x}{x^2+\sec x}$ (d) None of these

165. If $\sin(xy)+\frac{x}{y}=x^2-y$, then $\frac{dy}{dx}=$

(a) $\frac{y[2xy-y^2\cos(xy)-1]}{xy^2\cos(xy)+y^2-x}$ (b) $\frac{[2xy-y^2\cos(xy)-1]}{xy^2\cos(xy)+y^2-x}$ (c) $-\frac{y[2xy-y^2\cos(xy)-1]}{xy^2\cos(xy)+y^2-x}$ (d) None of these

166. If $3\sin(xy)+4\cos(xy)=5$, then $\frac{dy}{dx}=$ **[EAMCET 1994]**

(a) $-\frac{y}{x}$ (b) $\frac{3\sin(xy)+4\cos(xy)}{3\cos(xy)-4\sin(xy)}$ (c) $\frac{3\cos(xy)+4\sin(xy)}{4\cos(xy)-3\sin(xy)}$ (d) None of these

167. If $x^2e^y+2xye^x+13=0$, then $\frac{dy}{dx}=$ **[Rajasthan PET 1987]**

(a) $\frac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$ (b) $\frac{2xe^{x-y}+2y(x+1)}{x(xe^{y-x}+2)}$ (c) $-\frac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$ (d) None of these

168. If $\sin y = x\sin(a+y)$, then $\frac{dy}{dx} =$ **[Karnataka CET 2000; UPSEAT 2001]**

(a) $\frac{\sin^2(a+y)}{\sin(a+2y)}$ (b) $\frac{\sin^2(a+y)}{\cos(a+2y)}$ (c) $\frac{\sin^2(a+y)}{\sin a}$ (d) $\frac{\sin^2(a+y)}{\cos a}$

169. If $y = x^x$, then $\frac{dy}{dx} =$ **[AISSE 1984; DSSE 1982; MNR 1979; SCRA 1996; Rajasthan PET 1996; Kerala (Engg.) 2002]**

(a) $x^x \log ex$ (b) $x^x\left(1+\frac{1}{x}\right)$ (c) (1 + log x) (d) $x^x \log x$

170. If $y^x + x^y = a^b$, then $\frac{dy}{dx} =$

(a) $-\frac{y\,x^{y-1}+y^x\log y}{x\,y^{x-1}+x^y\log x}$ (b) $\frac{y\,x^{y-1}+y^x\log y}{x\,y^{x-1}+x^y\log x}$ (c) $-\frac{y\,x^{y-1}+y^x}{x\,y^{x-1}+x^y}$ (d) $\frac{y\,x^{y-1}+y^x}{x\,y^{x-1}+x^y}$

171. If $y = \sqrt{\frac{(x-a)(x-b)}{(x-c)(x-d)}}$, then $\frac{dy}{dx} =$

(a) $\frac{y}{2}\left[\frac{1}{x-a}+\frac{1}{x-b}-\frac{1}{x-c}-\frac{1}{x-d}\right]$ (b) $y\left[\frac{1}{x-a}+\frac{1}{x-b}-\frac{1}{x-c}-\frac{1}{x-d}\right]$

(c) $\frac{1}{2}\left[\frac{1}{x-a}+\frac{1}{x-b}-\frac{1}{x-c}-\frac{1}{x-d}\right]$ (d) None of these

172. $\frac{d}{dx}(x^{\log_e x}) =$ **[MP PET 1993]**

(a) $2x^{(\log_e x-1)}.\log_e x$ (b) $x^{(\log_e x-1)}$ (c) $\frac{2}{x}\log_e x$ (d) $x^{(\log_e x-1)}.\log_e x$

173. If $x^y = y^x$, then $\frac{dy}{dx} =$ **[DSSE 1986; MP PET 1997]**

(a) $\frac{y(x\log_e y+y)}{x(y\log_e x+x)}$ (b) $\frac{y(x\log_e y-y)}{x(y\log_e x-x)}$ (c) $\frac{x(x\log_e y-y)}{y(y\log_e x-x)}$ (d) $\frac{x(x\log_e y+y)}{y(y\log_e x+x)}$

174. If $y = x^{\sin x}$, then $\frac{dy}{dx} =$ **[DSSE 1983, 84]**

(a) $\frac{x\cos x.\log x+\sin x}{x}.x^{\sin x}$ (b) $\frac{y[x\cos x.\log x+\cos x]}{x}$

(c) $y[x\sin x.\log x+\cos x]$ (d) None of these

175. $\frac{d}{dx}\{(\sin x)^x\} =$ **[DSSE 1985, 87; AISSE 1983]**

(a) $\left[\frac{x\cos x+\sin x\log\sin x}{\sin x}\right]$ (b) $(\sin x)^x\left[\frac{x\cos x+\sin x\log\sin x}{\sin x}\right]$

(c) $(\sin x)^x\left[\frac{x\sin x+\sin x\log\sin x}{\sin x}\right]$ (d) None of these

176. If $2^x + 2^y = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$ is **[Karnataka CET 2000]**

(a) 0 (b) – 1 (c) 1 (d) 2

177. If $x^y = e^{x-y}$, then $\frac{dy}{dx} =$ **[MP PET 1987; MNR 1984; Roorkee 1954; Ranchi BIT 1991; Rajasthan PET 2000]**

(a) $\log x.[\log(ex)]^{-2}$ (b) $\log x[\log(ex)]^2$ (c) $\log x.(\log x)^2$ (d) None of these

178. $\frac{d}{dx}\{(\sin x)^{\log x}\} =$ **[DSSE 1984]**

(a) $(\sin x)^{\log x}\left[\frac{1}{x}\log \sin x + \cot x\right]$ (b) $(\sin x)^{\log x}\left[\frac{1}{x}\log \sin x + \cot x \log x\right]$

(c) $(\sin x)^{\log x}\left[\frac{1}{x}\log \sin x + \cot x\right]$ (d) None of these

179. $y = (\tan x)^{(\tan x)^{\tan x}}$, then at $x = \frac{\pi}{4}$, the value of $\frac{dy}{dx} =$ **[West Bengal JEE 1990]**

(a) 0 (b) 1 (c) 2 (d) None of these

180. If $x^p y^q = (x+y)^{p+q}$, then $\frac{dy}{dx} =$ **[Rajasthan PET 1999; UPSEAT 2001]**

(a) $\frac{y}{x}$ (b) $-\frac{y}{x}$ (c) $\frac{x}{y}$ (d) $-\frac{x}{y}$

181. If $y = (\tan x)^{\cot x}$, then $\frac{dy}{dx} =$

(a) $y \operatorname{cosec}^2 x (1 - \log \tan x)$ (b) $y \operatorname{cosec}^2 x (1 + \log \tan x)$ (c) $y \operatorname{cosec}^2 x (\log \tan x)$ (d) None of these

182. If $y = \frac{e^x \log x}{x^2}$, then $\frac{dy}{dx} =$ **[AI CBSE 1982]**

(a) $\frac{e^x[1+(x+2)\log x]}{x^3}$ (b) $\frac{e^x[1-(x-2)\log x]}{x^4}$ (c) $\frac{e^x[1-(x-2)\log x]}{x^3}$ (d) $\frac{e^x[1+(x-2)\log x]}{x^3}$

183. If $y = \frac{e^{2x}\cos x}{x \sin x}$, then $\frac{dy}{dx} =$

(a) $\frac{e^{2x}[(2x-1)\cot x - x\operatorname{cosec}^2 x]}{x^2}$ (b) $\frac{e^{2x}[(2x+1)\cot x - x\operatorname{cosec}^2 x]}{x^2}$

(c) $\frac{e^{2x}[(2x-1)\cot x + x\operatorname{cosec}^2 x]}{x^2}$ (d) None of these

184. If $y = \frac{\sqrt{x}(2x+3)^2}{\sqrt{x+1}}$, then $\frac{dy}{dx} =$ **[AISSE 1986]**

(a) $y\left[\frac{1}{2x}+\frac{4}{2x+3}-\frac{1}{2(x+1)}\right]$ (b) $y\left[\frac{1}{3x}+\frac{4}{2x+3}+\frac{1}{2(x+1)}\right]$ (c) $y\left[\frac{1}{3x}+\frac{4}{2x+3}+\frac{1}{(x+1)}\right]$ (d)

185. If $y = \frac{2(x-\sin x)^{3/2}}{\sqrt{x}}$, then $\frac{dy}{dx} =$

(a) $\frac{2(x-\sin x)^{3/2}}{\sqrt{x}}\left[\frac{3}{2}.\frac{1-\cos x}{1-\sin x}-\frac{1}{2x}\right]$ (b) $\frac{2(x-\sin x)^{3/2}}{\sqrt{x}}\left[\frac{3}{2}.\frac{1-\cos x}{x-\sin x}-\frac{1}{2x}\right]$

(c) $\frac{2(x-\sin x)^{1/2}}{\sqrt{x}}\left[\frac{3}{2}.\frac{1-\cos x}{x-\sin x}-\frac{1}{2x}\right]$ (d) None of these

186. $\frac{d}{dx}[(x-2)^x] =$ **[Rajasthan PET 1992]**

(a) $(x-2)^x[x+\log(x-2)]$ (b) $(x-2)^{x-1}[(x-2)\log(x-2)+x]$ (c) $(x-2)^{x-1}[x+\log(x-2)]$ (d) None of these

187. The derivative of x^{a^x} is

(a) $x^{a^x}\left[\frac{a^x}{x}+a^x\log a\log x\right]$ (b) $x^{a^x}[a^x+xa^x\log x]$ (c) $x^{a^x}[xa^x+a^x\log x]$ (d) None of these

188. If $x=a\sin 2\theta(1+\cos 2\theta), y=b\cos 2\theta(1-\cos 2\theta)$, then $\frac{dy}{dx}=$ **[Kurukshetra CEE 1998]**

(a) $\frac{b\tan\theta}{a}$ (b) $\frac{a\tan\theta}{b}$ (c) $\frac{a}{b\tan\theta}$ (d) $\frac{b}{a\tan\theta}$

189. If $x=a\left(\cos t+\log\tan\frac{t}{2}\right)$, $y=a\sin t$, then $\frac{dy}{dx}=$ **[Rajasthan PET 1997; MP PET 2001]**

(a) $\tan t$ (b) $-\tan t$ (c) $\cot t$ (d) $-\cot t$

190. If $x=\sin^{-1}(3t-4t^3)$ and $y=\cos^{-1}(\sqrt{1-t^2})$, then $\frac{dy}{dx}$ is equal to

(a) 1/2 (b) 2/5 (c) 3/2 (d) 1/3

191. If $x=\frac{2t}{1+t^2}$, $y=\frac{1-t^2}{1+t^2}$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1999]**

(a) $\frac{2t}{t^2+1}$ (b) $\frac{2t}{t^2-1}$ (c) $\frac{2t}{1-t^2}$ (d) None of these

192. If $x=\frac{3at}{1+t^3}$, $y=\frac{3at^2}{1+t^3}$, then $\frac{dy}{dx}=$

(a) $\frac{t(2+t^3)}{1-2t^3}$ (b) $\frac{t(2-t^3)}{1-2t^3}$ (c) $\frac{t(2+t^3)}{1+2t^3}$ (d) $\frac{t(2-t^3)}{1+2t^3}$

193. If $x = a(t+\sin t)$ and $y = a(1-\cos t)$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1996; MP PET 2002]**

(a) $\tan(t/2)$ (b) $\cot(t/2)$ (c) $\tan 2t$ (d) $\tan t$

194. If $x=a\cos^4\theta$, $y=a\sin^4\theta$, then $\frac{dy}{dx}$ at $\theta=\frac{3\pi}{4}$ **[Kerala (Engg.) 2002]**

(a) – 1 (b) 1 (c) $-a^2$ (d) a^2

195. If $x=2\cos t-\cos 2t$, $y=2\sin t-\sin 2t$, then at $t=\frac{\pi}{4}$, $\frac{dy}{dx}=$

(a) $\sqrt{2}+1$ (b) $\sqrt{2+1}$ (c) $\frac{\sqrt{2+1}}{2}$ (d) None of these

196. If $\tan y=\frac{2t}{1-t^2}$ and $\sin x=\frac{2t}{1+t^2}$, then $\frac{dy}{dx}=$ **[Rajasthan PET 1994]**

(a) $\frac{2}{1+t^2}$ (b) $\frac{1}{1+t^2}$ (c) 1 (d) 2

197. If $x=at^2$, $y = 2at$ then $\frac{dy}{dx}$ at $t = 2$ **[Rajasthan PET 1992]**

(a) 2 (b) 4 (c) 1/2 (d) 1/4

198. If $x=t^2+t+1$ and $y=\sin\frac{\pi}{2}t+\cos\frac{\pi}{2}t$ then at $t=1$ $\frac{dy}{dx}$ equals

(a) $-\pi/6$ (b) $\pi/2$ (c) $-\pi/4$ (d) $\pi/3$

199. If $y=e^{x+e^{x+e^{x+\dots\infty}}}$, then $\frac{dy}{dx}=$

(a) $\frac{y}{1-y}$ (b) $\frac{1}{1-y}$ (c) $\frac{y}{1+y}$ (d) $\frac{y}{y-1}$

200. If $y=(\sin x)^{(\sin x)^{(\sin x)......\infty}}$, then $\frac{dy}{dx}=$

(a) $\frac{y^2 \cot x}{1-y\log \sin x}$ (b) $\frac{y^2 \cot x}{1+y\log \sin x}$ (c) $\frac{y \cot x}{1-y\log \sin x}$ (d) $\frac{y \cot x}{1+y\log \sin x}$

201. The differential equation satisfied by the function $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+......\infty}}}$, is **[MP PET 1998]**

(a) $(2y-1)\frac{dy}{dx}-\sin x=0$ (b) $(2y-1)\cos x+\frac{dy}{dx}=0$

(c) $(2y-1)\cos x-\frac{dy}{dx}-=0$ (d) $(2y-1)\frac{dy}{dx}-\cos x=0$

202. If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+......\infty}}}$, then $\frac{dy}{dx}=$

(a) $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$ (c) $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

203. If $y=\sqrt{f(x)+\sqrt{f(x)+\sqrt{f(x)+......}}}$, then the value of (2y – 1) $\frac{dy}{dx}$ is

(a) $f(x)$ (b) $f'(x)$ (c) $2f'(x)$ (d) None of these

Advance Level

204. If $x^2+y^2=t-\frac{1}{t}$, $x^4+y^4=t^2+\frac{1}{t^2}$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1999]**

(a) $1/xy^3$ (b) $1/x^3y$ (c) $-1/x^3y$ (d) $-1/xy^3$

205. If $f'(x)=\sin(\log x)$ and $y=f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}=$ **[BIT Ranchi 1986]**

(a) $\frac{9\cos(\log x)}{x(3-2x)^2}$ (b) $\frac{9\cos\left(\log\frac{2x+3}{3-2x}\right)}{x(3-2x)^2}$ (c) $\frac{9\sin\left(\log\frac{2x+3}{3-2x^2}\right)}{(3-2x)^2}$ (d) None of these

206. $\frac{dy}{dx}$ of $\log(xy)=x^2+y^2$ is

(a) $\frac{y(2x^2-1)}{x(1-2y^2)}$ (b) $\frac{y(2x^2+1)}{x(1+2y^2)}$ (c) $\frac{x(2x^2-1)}{y(2y^2-1)}$ (d) $\frac{y(2x^2-1)}{x(2y^2-1)}$

207. $(x-y)e^{x/(x-y)}=k$, then

(a) $(y-2x)\frac{dy}{dx}+3x-2y=0$ (b) $y\frac{dy}{dx}+x-2y=0$ (c) $a\left(y\frac{dy}{dx}+x-2y\right)=1$ (d) None of these

208. If $y=(x^x)^x$, then $\frac{dy}{dx}=$

(a) $(x^x)^x(1+2\log x)$ (b) $(x^x)^x(1+\log x)$ (c) $x(x^x)^x(1+2\log x)$ (d) $x(x^x)^x(1+\log x)$

209. If $y=(x\log x)^{\log\log x}$, then $\frac{dy}{dx}=$ **[Roorkee 1981]**

(a) $(x\log x)^{\log\log x}\left\{\frac{1}{x\log x}(\log x+\log\log x)+(\log\log x)\left(\frac{1}{x}+\frac{1}{x\log x}\right)\right\}$

(b) $(x\log x)^{x\log x}\log\log x\left[\frac{2}{\log x}+\frac{1}{x}\right]$

(c) $(x\log x)^{x\log x}\frac{\log\log x}{x}\left[\frac{1}{\log x}+1\right]$

(d) None of these

210. If $y=\left(1+\frac{1}{x}\right)^x$, then $\frac{dy}{dx}=$ **[BIT Ranchi 1992]**

(a) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)-\frac{1}{1+x}\right]$ (b) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)\right]$

(c) $\left(x+\frac{1}{x}\right)^x\left[\log(x-1)-\frac{x}{1+x}\right]$ (d) $\left(x+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)+\frac{1}{1+x}\right]$

211. If $y=x^{(x^x)}$, then $\frac{dy}{dx}=$ **[AISSE 1989]**

(a) $y[x^x(\log ex).\log x+x^x]$ (b) $y[x^x(\log ex).\log x+x]$ (c) $y[x^x(\log ex).\log x+x^{x-1}]$ (d) $y[x^x(\log_e x).\log x+x^{x-1}]$

212. If $y=\frac{a^{\cos^{-1}x}}{1+a^{\cos^{-1}x}}$ and $z=a^{\cos^{-1}x}$, then $\frac{dy}{dz}=$ **[MP PET 1994]**

(a) $\frac{1}{1+a^{\cos^{-1}x}}$ (b) $-\frac{1}{1+a^{\cos^{-1}x}}$ (c) $\frac{1}{(1+a^{\cos^{-1}x})^2}$ (d) None of these

213. Let the function $y=f(x)$ be given by $x=t^5-5t^3-20t+7$ and $y=4t^3-3t^2-18t+3$, where $t\in(-2,2)$. Then $f'(x)$ at $t=1$ is

(a) $\frac{5}{2}$ (b) $\frac{2}{5}$ (c) $\frac{7}{5}$ (d) None of these

214. If $y=\sqrt{x}^{\sqrt{x}^{\sqrt{x}......\infty}}$, then $\frac{dy}{dx}=$

(a) $\frac{y^2}{2x-2y\log x}$ (b) $\frac{y^2}{2x+\log x}$ (c) $\frac{y^2}{2x+2y\log x}$ (d) None of these

215. If $y=x+\cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x+.....}}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{y}{2y-x}$ (b) $\frac{y}{2y+x}$ (c) $\frac{y}{y-2x}$ (d) $\frac{y}{y+2x}$

216. If $y=\cfrac{x}{a+\cfrac{x}{b+\cfrac{x}{a+\cfrac{x}{b+.........}}}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{b}{a(b+2y)}$ (b) $\frac{b}{b+2y}$ (c) $\frac{a}{b(b+2y)}$ (d) None of these

217. If $y=\cfrac{\sin x}{1+\cfrac{\cos x}{1+\cfrac{\sin x}{1+\cos x.......\infty}}}$, then $\frac{dy}{dx}=$

(a) $\frac{(1+y)\cos x+\sin x}{1+2y+\cos x-\sin x}$ (b) $\frac{(1+y)\cos x-\sin x}{1+2y+\cos x+\sin x}$ (c) $\frac{(1+y)\cos x+\sin x}{1+2y+\cos x+\sin x}$ (d) None of these

218. If $f(x)=\frac{1}{1-x}$, then the derivative of the composite function $f[f\{f(x)\}]$ is equal to **[Orissa JEE 2003]**

(a) 0 (b) 1/2 (c) 1 (d) 2

219. If $u=f(x^3), v=g(x^2), f'(x)=\cos x$ and $g'(x)=\sin x$ then $\frac{du}{dv}$ is

(a) $\frac{3}{2}x.\cos x^3.\text{cosec}\, x^2$ (b) $\frac{2}{3}\sin x^3.\sec x^2$ (c) $\tan x$ (d) None of these

220. Let $f(x)=e^x, g(x)=\sin^{-1}x$ and $h(x)=f(g(x))$, then $h'(x)/h(x)=$

(a) $e^{\sin^{-1}x}$ (b) $1/\sqrt{1-x^2}$ (c) $\sin^{-1}x$ (d) $1/(1-x^2)$

Differentiation of a Function with Respect to Another Function

Basic Level

221. The derivative of $\sin^2 x$ with respect to $\cos^2 x$ is **[DCE 2002]**

(a) $\tan^2 x$ (b) $\tan x$ (c) $-\tan x$ (d) None of these

222. The differential of e^{x^3} with respect to $\log x$ is **[KCET 2002]**

(a) e^{x^3} (b) $3x^2e^{x^3}$ (c) $3x^3e^{x^3}$ (d) $3x^2e^{x^3}+3x^2$

223. The differential coefficient of x^6 with respect to x^3 is **[EAMCET 1988; UPSEAT 2000]**

(a) $5x^2$ (b) $3x^3$ (c) $5x^5$ (d) $2x^3$

224. The rate of change of $\sqrt{x^2+16}$ with respect to $\frac{x}{x-1}$ at $x = 3$, will be **[MP PET 1987]**

(a) $-\frac{24}{5}$ (b) $\frac{24}{5}$ (c) $\frac{12}{5}$ (d) $-\frac{12}{5}$

225. Differential coefficient of $\sin^{-1}\frac{1-x}{1+x}$ *w.r.t.* $\sqrt{x}$ is **[Roorkee 1984]**

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{\sqrt{x}}{\sqrt{1-x}}$ (c) 1 (d) None of these

226. Differential coefficient of $\sec^{-1}\frac{1}{2x^2-1}$ *w.r.t.* $\sqrt{1-x^2}$ at $x=\frac{1}{2}$ is

(a) 2 (b) 4 (c) 6 (d) 1

227. Differential coefficient of $\sin^{-1}x$ *w.r.t.* $\cos^{-1}\sqrt{1-x^2}$ is **[MNR 1983; AMU 2002]**

(a) 1 (b) $\frac{1}{1+x^2}$ (c) 2 (d) None of these

228. The differential coefficient of $\tan^{-1}\sqrt{x}$ with respect to $\sqrt{x}$ is

(a) $\frac{1}{\sqrt{1+x}}$ (b) $\frac{1}{2x\sqrt{1+x}}$ (c) $\frac{1}{2\sqrt{x(1+x)}}$ (d) $\frac{1}{1+x}$

229. Derivative of $\sec^{-1}\left\{\frac{1}{2x^2-1}\right\}$ *w.r.t.* $\sqrt{1+3x}$ at $x=-\frac{1}{3}$ is **[EAMCET 1991]**

(a) 0 (b) 1/2 (c) 1/3 (d) None of these

230. Differential coefficient of $\cos^{-1}(\sqrt{x})$ with respect to $\sqrt{(1-x)}$ is

(a) $\sqrt{x}$ (b) $-\sqrt{x}$ (c) $\frac{1}{\sqrt{x}}$ (d) $-\frac{1}{\sqrt{x}}$

231. Differential coefficient of $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ *w.r.t.* $\cos^{-1}(x^2)$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) 0

232. If $u=\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ and $v=2\tan^{-1}x$, then $\frac{du}{dv}$ is equal to

(a) 4 (b) 1 (c) 1/4 (d) – 1/4

233. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ *w.r.t.* $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is **[Karnataka CET 2000]**

(a) – 1 (b) 1 (c) 2 (d) 4

234. Differential coefficient of $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ *w.r.t.* $\sin^{-1}x$, is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{3}{2}$

235. The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ *w.r.t.* $\cot^{-1}\left(\frac{1-3x^2}{3x-x^2}\right)$ is **[Karnataka CET 2003]**

(a) 1 (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

236. The differential coefficient of $e^{\sin^{-1}x}$ with respect to $\sin^{-1}x$ is

(a) $\cos^{-1}x$ (b) $e^{\cos^{-1}x}$ (c) $e^{\sin^{-1}x}$ (d) $\sin^{-1}x$

Advance Level

237. Differential coefficient of $\frac{\tan^{-1}x}{1+\tan^{-1}x}$ *w.r.t.* $\tan^{-1}x$ is

(a) $\frac{1}{1+\tan^{-1}x}$ (b) $\frac{-1}{1+\tan^{-1}x}$ (c) $\frac{1}{(1+\tan^{-1}x)^2}$ (d) $\frac{-1}{2(1+\tan^{-1}x)^2}$

238. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = 0$, is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

239. Differentiation of $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}(2x\sqrt{1-x^2})$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) – 1

240. Differentiation of $\sin^{-1}(2ax\sqrt{1-a^2x^2})$ with respect to $\sqrt{1-a^2x^2}$ is

(a) 2 (b) ax (c) $\frac{2}{ax}$ (d) $-\frac{2}{ax}$

241. Differentiation of $\tan^{-1}\left(\frac{1+ax}{1-ax}\right)$ with respect to $\sqrt{1+a^2x^2}$ is

(a) $\frac{1}{ax\sqrt{1+ax}}$ (b) $\frac{1}{\sqrt{1+ax}}$ (c) $\frac{1}{ax\sqrt{1+a^2x^2}}$ (d) $\frac{1}{ax\sqrt{1-a^2x^2}}$

242. The value of derivative of $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ *w.r.t.* to $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ at $x=\frac{1}{2}$ equals

(a) 1 (b) – 1 (c) 0 (d) None of these

Successive Differentiation or Higher Order Derivatives

Basic Level

243. If $y=(x^2-1)^m$, then the $(2m)^{th}$ differential coefficient of y is

(a) m (b) $(2m)!$ (c) $2m$ (d) $m!$

244. The n^{th} derivative of xe^x vanishes when **[AMU 1999]**

(a) $x = 0$ (b) $x = -1$ (c) $x = -n$ (d) $x = n$

245. If $x^p y^q = (x+y)^{p+q}$, then $\frac{d^2y}{dx^2}=$ **[West Bengal JEE 1992]**

(a) 0 (b) 1 (c) 2 (d) None of these

246. If $y = A\cos nx + B\sin nx$, then $\frac{d^2y}{dx^2}=$ **[Karnataka CET 1996]**

(a) n^2y (b) $-y$ (c) $-n^2y$ (d) None of these

247. If $x = a\sin\theta$ and $y = b\cos\theta$, then $\frac{d^2y}{dx^2}$ is **[UPSEAT 2002]**

(a) $\frac{a}{b^2}\sec^2\theta$ (b) $\frac{-b}{a}\sec^2\theta$ (c) $\frac{-b}{a^2}\sec^3\theta$ (d) $\frac{-b}{a^2\sec^3\theta}$

248. If $y = a\cos(\log x)+\sin(\log x)$, then

(a) $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$ (b) $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-y=0$ (c) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$ (d) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

249. If $e^y + xy = e$, then the value of $\frac{d^2y}{dx^2}$ for $x = 0$, is **[Kurukshetra CEE 2002]**

(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$ (c) $\frac{1}{e^3}$ (d) None of these

250. If $y=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-......$, then $\frac{d^2y}{dx^2}=$ **[Karnataka CET 2003]**

(a) x (b) $-x$ (c) $-y$ (d) y

251. If $y = ax^{n+1}+bx^{-n}$, then $x^2\frac{d^2y}{dx^2}=$

(a) $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

252. If $y = a + bx^2$; a, b arbitrary constants, then **[EAMCET 1994]**

(a) $\frac{d^2y}{dx^2} = 2xy$ (b) $x\frac{d^2y}{dx^2} = \frac{dy}{dx}$ (c) $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$ (d) $x\frac{d^2y}{dx^2} = 2xy$

253. If $y = x\log\left(\frac{x}{a+bx}\right)$, then $x^3\frac{d^2y}{dx^2} =$ **[West Bengal JEE 1991; Roorkeee 1976]**

(a) $x\frac{dy}{dx} - y$ (b) $\left(x\frac{dy}{dx} - y\right)^2$ (c) $y\frac{dy}{dx} - x$ (d) $\left(y\frac{dy}{dx} - x\right)^2$

254. $\frac{d^2}{dx^2}(2\cos x\cos 3x) =$ **[Rajasthan PET 2003]**

(a) $2^2(\cos 2x + 2^2\cos 4x)$ (b) $2^2(\cos 2x - 2^2\cos 4x)$ (c) $2^2(-\cos 2x + 2^2\cos 4x)$ (d) $-2^2(\cos 2x + 2^2\cos 4x)$

255. If $x = t^2$, $y = t^3$, then $\frac{d^2y}{dx^2} =$

(a) 3/2 (b) 3/(4t) (c) 3/(2t) (d) 3t/2

256. If $y = ae^{mx} + be^{-mx}$, then $\frac{d^2y}{dx^2} - m^2y =$ **[MP PET 1987]**

(a) $m^2(ae^{mx} - be^{-mx})$ (b) 1 (c) 0 (d) None of these

257. If $y = x^2e^{mx}$, where m is a constant, then $\frac{d^3y}{dx^3} =$ **[MP PET 1987]**

(a) $me^{mx}(m^2x^2 + 6mx + 6)$ (b) $2m^3xe^{mx}$ (c) $me^{mx}(m^2x^2 + 2mx + 2)$ (d) None of these

258. If f be a polynomial, then the second derivative of $f(e^x)$ is

(a) $f'(e^x)$ (b) $f''(e^x)e^x + f'(e^x)$ (c) $f''(e^x)e^{2x} + f''(e^x)$ (d) $f''(e^x)e^{2x} + f'(e^x)e^x$

259. If $y = ae^x + be^{-x} + c$ where a, b, c are parameters then $y''' =$

(a) y (b) y' (c) 0 (d) y''

260. If y = a cos(log x) + b sin(log x) where a, b are parameters then $x^2y'' + xy' =$ **[EAMCET 2002]**

(a) y (b) – y (c) 2y (d) – 2y

261. If $y = x^3\log\log_e(1+x)$ then $y''(0)$ equals **[AMU 1999]**

(a) 0 (b) – 1 (c) $6\log_e 2$ (d) 6

262. $\frac{d^2x}{dy^2}$ is equal to **[AMU 2001]**

(a) $\frac{1}{(dy/dx)^2}$ (b) $\frac{(d^2y/dx^2)}{(dy/dx)^2}$ (c) $\frac{d^2y}{dx^2}$ (d) $\frac{(-d^2y/dx^2)}{(dy/dx)^2}$

263. If $x = e^t\sin t$, $y = e^t\cos t$, t is a parameter, then $\frac{d^2y}{dx^2}$ at (1, 1) is equal to **[AMU 2001]**

(a) –1/2 (b) –1/4 (c) 0 (d) 1/2

264. $\frac{d^n}{dx^n}(\sin 2x) =$

(a) $\sin\left(\frac{n\pi}{2}+x\right)$ (b) $2^n \sin\left(\frac{n\pi}{2}+2x\right)$ (c) $2^n \sin\left(\frac{\pi}{2}+2x\right)$ (d) None of these

265. $\frac{d^n}{dx^n}(\log x) =$ **[Rajasthan PET 2002]**

(a) $\frac{(n-1)!}{x^n}$ (b) $\frac{n!}{x^n}$ (c) $\frac{(n-2)!}{x^n}$ (d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

266. $\frac{d^n}{dx^n}(e^{2x}+e^{-2x}) =$

(a) $e^{2x}+(-1)^n e^{-2x}$ (b) $2^n(e^{2x}-e^{-2x})$ (c) $2^n[e^{2x}+(-1)^n e^{-2x}]$ (d) None of these

267. If $y = \sin x \sin 3x$, then $y_n =$

(a) $\frac{1}{2}\left[\cos\left(2x+n\frac{\pi}{2}\right)-\cos\left(4x+n\frac{\pi}{2}\right)\right]$ (b) $\frac{1}{2}\left[2^n\cos\left(2x+n\frac{\pi}{2}\right)-4^n\cos\left(4x+n\frac{\pi}{2}\right)\right]$

(c) $\frac{1}{2}\left[4^n\cos\left(4x+n\frac{\pi}{2}\right)-2^n\cos\left(2x+n\frac{\pi}{2}\right)\right]$ (d) None of these

268. The n^{th} derivative of $\frac{x}{1-x}$ is

(a) $\frac{(-1)^n n!}{(1-x)^{n+1}}$ (b) $\frac{n!}{(1-x)^{n+1}}$ (c) $\frac{(-1)^n}{(1-x)^{n+1}}$ (d) $\frac{1}{(1-x)^{n+1}}$

269. If $y = \sin^2 x$, then value of y_n is

(a) $2^n\cos\left(2x+\frac{n\pi}{2}\right)$ (b) $-2^n\cos\left(2x+\frac{n\pi}{2}\right)$ (c) $-2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$ (d) None of these

270. If $y = \sin 2x \cos 2x$, then value of y_n is

(a) $2^{2n-1}\sin\left(4x+\frac{n\pi}{2}\right)$ (b) $2^{2n}\sin\left(4x+\frac{n\pi}{2}\right)$ (c) $2^{2n-1}\cos\left(4x+\frac{n\pi}{2}\right)$ (d) None of these

271. If $y = e^{6-5x}$, then the value of y_n is

(a) $5^n e^{6-5x}$ (b) $(-5)^n e^{6-5x}$ (c) $5^{n-1}e^{6-5x}$ (d) $(-5)^{n-1}e^{6-5x}$

272. If $y = 8^x$, then the value of y_n is

(a) $\frac{8^x}{\log_e 8}$ (b) $\frac{8^x}{(\log_e 8)^n}$ (c) $8^x \log_e 8$ (d) $8^x(\log_e 8)^n$

273. $D^n[f(ax+b)]$ is equal to

(a) $n!\,f_n(ax+b)$ (b) $a^n\,f_n(ax+b)$ (c) $(n-1)!\,a^n\,f_n(ax+b)$ (d) 0

274. If $y = x^{n-1}\log x$, then which of the following statement is true

(a) $xy_n = n!$ (b) $xy_n = (n-1)!$ (c) $xy_n = (n-2)!$ (d) $x^2y_n = n!$

Advance Level

275. If $x = f_1(t)$ and $y = f_2(t)$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{f_1' f_2'' - f_2' f_1''}{(f_1')^2}$ (b) $\frac{f_1' f_2'' - f_2' f_1''}{(f_1')^3}$ (c) $\frac{f_1''(t)}{f_2''(t)}$ (d) $\frac{-f_1''(t)}{f_2''(t)}$

276. If $y^2 = p(x)$ is a polynomial of degree three, then $2\frac{d}{dx}\left\{y^3 . \frac{d^2y}{dx^2}\right\} =$ **[IIT 1988; Rajasthan PET 2000]**

(a) $p'''(x) + p'(x)$ (b) $p''(x).p'''(x)$ (c) $p(x).p'''(x)$ (d) Constant

277. If $x = a\cos\theta$, $y = b\sin\theta$, then $\frac{d^3y}{dx^3}$ is equal to

(a) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot^4\theta$ (b) $\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta$ (c) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta$ (d) None of these

278. $\frac{d^{20}y}{dx^{20}}(2\cos x\cos 3x) =$ **[EAMCET 1994]**

(a) $2^{20}(\cos 2x - 2^{20}\cos 4x)$ (b) $2^{20}(\cos 2x + 2^{20}\cos 4x)$ (c) $2^{20}(\sin 2x + 2^{20}\sin 4x)$ (d) $2^{20}(\sin 2x - 2^{20}\sin 4x)$

279. If $u = x^2 + y^2$ and $x = s + 3t$, $y = 2s - t$, then $\frac{d^2u}{ds^2} =$ **[Orissa JEE 2002]**

(a) 12 (b) 32 (c) 36 (d) 10

280. If $y = \sin x + e^x$, then $\frac{d^2x}{dy^2} =$ **[KCET 1999; UPSEAT 2001; Haryana CEE 2002]**

(a) $(-\sin x + e^x)^{-1}$ (b) $\frac{\sin x - e^x}{(\cos x + e^x)^2}$ (c) $\frac{\sin x - e^x}{(\cos x + e^x)^3}$ (d) $\frac{\sin x + e^x}{(\cos x + e^x)^3}$

281. If $x = at^2$, $y = 2at$, then $\frac{d^2y}{dx^2} =$ **[Karnataka CET 1993]**

(a) $-\frac{1}{t^2}$ (b) $\frac{1}{2at^3}$ (c) $-\frac{1}{t^3}$ (d) $-\frac{1}{2at^3}$

282. If $I_n = \frac{d^n}{dx^n}(x^n \log x)$, then $I_n - nI_{n-1} =$ **[EAMCET 2003]**

(a) n (b) $n - 1$ (c) $n\,!$ (d) $(n - 1)!$

283. If $y = (\sin^{-1} x)^2 + k\sin^{-1} x$ then which is true

(a) $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$ (b) $(1 + x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$ (c) $(1 - x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$ (d) $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$

284. If $y = e^{\tan^{-1} x}$ then which is true

(a) $(1 + x^2)y_2 + (2x - 1)y_1 = 0$ (b) $(1 + x^2)y_2 + (2x + 1)y_1 = 0$ (c) $(1 + x^2)y_2 - (2x - 1)y_1 = 0$ (d) $(1 + x^2)y_2 - (2x + 1)y_1 = 0$

285. The function $u = e^x \sin x$, $v = e^x \cos x$ satisfy the equation

(a) $v\frac{du}{dv} = u\frac{dv}{dx} + u^2 + v^2$ (b) $\frac{d^2u}{dx^2} = 2v$ (c) $\frac{d^2v}{dx^2} = -2u$ (d) None of these

286. If $x^2+y^2=a^2$ and $k=\frac{1}{a}$, then k is equal to

(a) $\frac{y''}{\sqrt{1+y'}}$ (b) $\frac{|y''|}{\sqrt{(1+y'^2)^3}}$ (c) $\frac{2y''}{\sqrt{1+y'^2}}$ (d) $\frac{y''}{2\sqrt{(1+y'^2)^3}}$

287. If $(a+bx)e^{y/x}=x$, then the value of $x^3\frac{d^2y}{dx^2}$ is

(a) $\left(y\frac{dy}{dx}-x\right)^2$ (b) $\left(x\frac{dy}{dx}-y\right)^2$ (c) $x\frac{dy}{dx}-y$ (d) None of these

288. If $y=[\log(x+\sqrt{x^2+1})]^2$ then which is correct

(a) $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=2$ (b) $(1+x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$ (c) $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$ (d) $(1+x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$

289. If $y=\frac{1}{x^2-a^2}$, then $\frac{d^2y}{dx^2}$ equals

(a) $\frac{3x^2+a^2}{(x^2-a^2)^3}$ (b) $\frac{3x^2+a^2}{(x^2-a^2)^4}$ (c) $\frac{2(3x^2+a^2)}{(x^2-a^2)^3}$ (d) $\frac{2(3x^2+a^2)}{(x^2-a^2)^4}$

290. If $y^{1/m}+y^{-1/m}=2x$, then $(x^2-1)y_2+xy_1$ is equal to

(a) m^2y (b) $-m^2y$ (c) $\pm m^2y$ (d) $\pm my$

291. If $y=(\sin^{-1}x)^2+(\cos^{-1}x)^2$, then $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}$ is equal to

(a) 4 (b) 3 (c) 1 (d) 0

292. If $y=e^{\sqrt{x}}+e^{-\sqrt{x}}$, then $xy_2+\frac{1}{2}y_1-\frac{1}{4}y$ is equal to

(a) 0 (b) 1 (c) – 1 (d) 2

293. If $y=\sin 2x$ then $\frac{d^6y}{dx^6}$ at $x=\frac{\pi}{2}$ is equal to

(a) – 64 (b) 0 (c) 64 (d) None of these

294. $\frac{d^n}{dx^n}\cos^2 x=$

(a) $2^{n-1}\cos\left(2x+\frac{\pi}{2}\right)$ (b) $2^{n-1}\cos\left(2x-\frac{\pi}{2}\right)$ (c) $2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$ (d) $2^{n-1}\cos\left(2x-\frac{n\pi}{2}\right)$

295. If $y=\cos^4 x$, then y_n is equal to

(a) $2^{2n-3}\cos\left(4x+\frac{n\pi}{2}\right)+2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$ (b) $2^{2n-3}\cos\left(2x+\frac{n\pi}{2}\right)+2^{n-1}\cos\left(4x+\frac{n\pi}{2}\right)$

(c) $\cos\left(4x+\frac{n\pi}{2}\right)+\cos\left(2x+\frac{n\pi}{2}\right)$ (d) None of these

296. If $y=\sin^2 x\sin 2x$ then y_n is equal to

(a) $2^{n-1}\sin\left(2x+\frac{n\pi}{2}\right)+4^{n-1}\sin\left(4x+\frac{n\pi}{2}\right)$ (b) $2^{n-1}\sin\left(2x+\frac{n\pi}{2}\right)+4^{n-1}\sin\left(4x+\frac{n\pi}{2}\right)$

(c) $2\sin\left(2x+\frac{n\pi}{2}\right)+\sin\left(4x+\frac{n\pi}{2}\right)$ (d) None of these

n^{th} Derivative Using Partial Fractions

Basic Level

297. n^{th} derivative of $\frac{1}{3x^2-5x+2}$ is

(a) $(-1)^n n!\left\{\frac{1}{(x-1)^{n+1}}+\frac{3^{n+1}}{(3x-2)^{n+1}}\right\}$ (b) $n!\left\{\frac{1}{(x-1)^{n+1}}-\frac{3^{n+1}}{(3x-2)^{n+1}}\right\}$

(c) $(-1)^n n!\left\{\frac{1}{(x-1)^{n+1}}-\frac{3^{n+1}}{(3x-2)^{n+1}}\right\}$ (d) None of these

298. n^{th} derivative of $\frac{1}{x^2+5x+6}$ is

(a) $(-1)^n n!\left[\frac{1}{(x+2)^{n+1}}+\frac{1}{(x+3)^{n+1}}\right]$ (b) $(-1)^n n!\left[\frac{1}{(x+3)^{n+1}}-\frac{1}{(x+2)^{n+1}}\right]$ (c) $(-1)^n n!\left[\frac{1}{(x+2)^{n+1}}-\frac{1}{(x+3)^{n+1}}\right]$ (d) None of these

Advance Level

299. n^{th} derivative of $\frac{2x+3}{5x+7}$

(a) $\frac{(-1)^n n!5^{n-1}}{(5x+7)^{n+1}}$ (b) $\frac{(-1)^n n!5^{n-1}}{(5x+7)^{n-1}}$ (c) $\frac{(-1)^n n!5^{n+1}}{(5x+7)^{n+1}}$ (d) $\frac{(-1)^n n!5^{n+1}}{(5x+7)^{n-1}}$

300. n^{th} derivative of $\frac{1}{x^2-a^2}$ is

(a) $\frac{(-1)^n n!}{2a}[(x-a)^{n+1}-(x+a)^{n-1}]$ (b) $\frac{(-1)^n n!}{2a}[(x+a)^{n+1}-(x-a)^{n+1}]$

(c) $\frac{(-1)^n n!}{2a}\left[\frac{1}{(x-a)^{n+1}}-\frac{1}{(x+a)^{n+1}}\right]$ (d) $\frac{(-1)^n n!}{2a}[(x-a)^{n+1}+(x+a)^{n+1}]$

Differentiation of Determinants

Basic Level

301. If $f(x)=\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$, then $f'(x)$ is

(a) x^2 (b) $6x$ (c) $6x^2$ (d) 1

302. If $f(x)=\begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta\sec x & \tan x & x \\ 1 & \tan x-\tan\theta & 0 \end{vmatrix}$, then $f'(\theta)$ is

(a) 0 (b) – 1 (c) Independent of θ (d) None of these

303. Let f, g, h and k be differentiable in (a, b) and F is defined as $F(x)=\begin{vmatrix} f(x) & g(x) \\ h(x) & k(x) \end{vmatrix}$ for all $x \in (a,b)$ then F' is given by

(a) $\begin{vmatrix} f & g \\ h & k \end{vmatrix}+\begin{vmatrix} f & g \\ h' & k \end{vmatrix}$ (b) $\begin{vmatrix} f' & g' \\ h & k \end{vmatrix}+\begin{vmatrix} f & g \\ h' & k' \end{vmatrix}$ (c) $\begin{vmatrix} f & g' \\ h & k' \end{vmatrix}+\begin{vmatrix} f' & g \\ h & k' \end{vmatrix}$ (d) $\begin{vmatrix} f & g \\ h' & k' \end{vmatrix}+\begin{vmatrix} f' & g \\ h & k \end{vmatrix}$

Advance Level

304. $f(x)=\begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$, here p is a constant, then $\dfrac{d^3 f(x)}{dx^3}$ is

(a) Proportional to x^2 (b) Proportional to x (c) Proportional to x^3 (d) A constant

305. If $y = \sin px$ and y_n is the n^{th} derivative of y, then $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ is equal to **[AMU 2002]**

(a) 1 (b) 0 (c) – 1 (d) None of these

Differentiation of Integral Functions

Basic Level

306. Let $f(t) = \log(t)$, then $\dfrac{d}{dx}\left(\int_{x^2}^{x^3} f(t)dt\right)$

(a) Has a value 0 when x = 0 (b) Has a value 0 when x = 1 and $x = \dfrac{4}{9}$

(c) Has a value $9e^2 - 4$ when $x = e$ (d) Has a differential coefficient $27e - 8$ for $x = e$

307. If $f(x) = \int_0^x t\sin t\, dt$, then $f'(x) =$

(a) $x \sin x$ (b) $x \cos x$ (c) $\sin x + \cos x$ (d) $x^2/2$

Advance Level

308. If $F(x)=\dfrac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\, dt$, then $F'(4)$ equals

(a) $32/9$ (b) $64/3$ (c) $64/9$ (d) None of these

Leibneitz's theorem

Basic Level

309. If $y = x \sin x$, then at $x = 0$ the value of y_{15} equal to

(a) 0 (b) – 15 (c) 15! (d) –(15)!

Advance Level

310. If $y = xe^x$ then the value of y_n is

(a) $(n+1)e^x$ (b) $(x+1)e^x$ (c) $(x+n)e^x$ (d) $(x-n)e^x$

Miscellaneous Problems

Basic Level

311. Given that $d/dx\, f(x) = f'(x)$. The relationship $f'(a+b) = f'(a) + f'(b)$ is valid if $f(x)$ is equal to

(a) x (b) x^2 (c) x^3 (d) x^4

312. $f(x)$ and $g(x)$ are two differentiable function on [0, 2] such that $f''(x) - g''(x) = 0, f'(1) = 2, g'(1) = 4, f(2) = 3, g(2) = 9$, then $f(x) - g(x)$ at $x = 3/2$ is

(a) 0 (b) 2 (c) 10 (d) – 5

313. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, then $\frac{y'}{y} =$ **[IIT 1998]**

(a) $\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$ (b) $\left(\frac{a}{a+x} + \frac{b}{b+x} + \frac{c}{c+x}\right)$ (c) $\frac{1}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$ (d) $\frac{1}{y}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$

314. If $y = \frac{1}{1+x^{a-b}+x^{a-c}} + \frac{1}{1+x^{b-a}+x^{b-c}} + \frac{1}{1+x^{c-a}+x^{c-b}}$, then $\frac{dy}{dx}$ equals

(a) $ax^{-1} + bx^{-1} + cx^{-1}$ (b) 0 (c) 1 (d) $a+b+c$

315. Let $f(x)$ be a polynomial function of the second degree. If $f(1) = f(-1)$ and a_1, a_2, a_3 are in A.P. then $f'(a_1), f'(a_2), f'(a_3)$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

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Answer Sheet

Assignment (Basic & Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	d	c	b	d	c	b	a	b	b	a	b	d	a	a	c	a	d	c	b
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
d	d	c	b	b	d	d	a	c	c	d	c	d	a	b	c	d	c	d	d
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
d	b	a	b	a	d	c	b	b	a	c	b	b	b	d	b	c	c	c	a
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
a	a	c	c	a	a	a	a	c	c	a	a	a	b	a	c	c	a	c	c
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
a	d	c	b	b	b	a	c	c	a	d	a	c	c	b	a	a	c	a	a
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
d	d	c	c	a	b	c	a	b	c	d	b	b	d	a	c	b	c	a	d
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
c	c	a	b	d	a	a	a	a	a	a	c	c	c	a	a	b	a	b	b
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
b	c	b	b	a	b	c	a	c	b	c	c	c	c	c	c	c	d	a	c
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
a	a	b	c	b	a	c	c	a	a	a	a	b	a	b	b	a	b	c	a
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
a	d	a	a	b	b	a	a	a	d	b	b	a	a	a	c	c	a	a	a
201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
d	c	b	b	d	a	b	c	a	a	c	c	b	d	b	a	b	c	a	b
221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
d	c	d	d	d	b	a	d	a	c	a	c	b	a	c	a	c	b	b	d
241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260
c	b	b	c	a	c	c	d	b	d	b	b	b	d	b	c	a	d	b	b
261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280
a	d	a	b	d	c	b	b	a	a	b	d	b	b	b	c	c	b	d	c
281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300
b	d	a	a	b	b	c	a	c	a	a	a	a	c	c	b	c	c	a	c
301	302	303	304	305	306	307	308	309	310	311	312	313	314	315					
c	c	b	d	b	b	a	a	d	c	b	d	c	b	a					